

# Binomial Theorem Notes

$$(a+b)^0 = 1$$

$$(a+b)^1 = 1a + 1b$$

$$(a+b)^2 = 1a^2 + 2ab + 1b^2$$

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$

$$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$$

\* All exponents add up to the original exponent.

\* The coefficients are determined by...  
 - Pascal's Triangle or...  
 use the Combinations Formula

Power  $n=0$  → Coefficients

$n=1$  → 1 1

$n=2$  → 1 2 1

$n=3$  → 1 3 3 1

$n=4$  → 1 4 6 4 1

$n=5$  → 1 5 10 10 5 1

$n=6$  → 1 6 15 20 15 6 1

$n=7$  → 1 7 21 35 35 21 7 1

$$nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \dots$$

apply

$$(x+y)^8$$

$$\binom{8}{0}x^8 + \binom{8}{1}x^7y + \binom{8}{2}x^6y^2 + \binom{8}{3}x^5y^3 + \binom{8}{4}x^4y^4 + \binom{8}{5}x^3y^5 + \binom{8}{6}x^2y^6 +$$

$$\binom{8}{7}xy^7 + \binom{8}{8}y^8$$

Solution next page.

IC-112  $(x+y)^8$

$$\begin{aligned} & \left(\frac{8!}{(8-0)!0!}\right)x^8 + \left(\frac{8!}{(8-1)!1!}\right)x^7y + \left(\frac{8!}{(8-2)!2!}\right)x^6y^2 + \left(\frac{8!}{(8-3)!3!}\right)x^5y^3 + \\ & \left(\frac{8!}{(8-4)!4!}\right)x^4y^4 + \left(\frac{8!}{(8-5)!5!}\right)x^3y^5 + \left(\frac{8!}{(8-6)!6!}\right)x^2y^6 + \left(\frac{8!}{(8-7)!7!}\right)xy^7 + \left(\frac{8!}{(8-8)!8!}\right)y^8 \end{aligned}$$

$$\boxed{x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 28x^2y^6 + 8xy^7 + y^8}$$

1st          2nd          3rd          4th          5th          6th          7th          8th

Next day practice Binomial Theorem.  
(worksheet)

Add to notes       $r = \text{term \#}$        $n = \text{exponent}$

to find the  $r^{\text{th}}$  term

- use combination formula to find the coefficient.
- determine the leading exponent

$$\boxed{(n-r)+1}$$

example:  $(x+y)^7$  ← 4<sup>th</sup> term

$$n=7 \quad r=4$$

$${}_7C_4 = \frac{7!}{(7-4)!4!} = 35$$

$$(7-4)+1 = 4$$

$$\boxed{35x^4y^3}$$

exponents must add up to original exponent

\* If it's a difference even terms are "-".