Multiplying

In arithmetic we wrote multiplication problems these two ways:

$$3 \times 5 = 15 \qquad \begin{array}{r} 3 \\ \times\, 5 \\ \hline 15 \end{array}$$

In algebra we show multiplication by using a **dot** or by using **parentheses**. Below are some examples.

$$3 \cdot 5 = 15 \qquad 3(5) = 15 \qquad (3)(5) = 15$$

Here are some multiplication problems for you to do:

$4 \cdot 3 = 12$	$10 \cdot 7 = 70$	$2 \cdot 4 = 8$
$3 \cdot 6 = 18$	$(8)(3) = 24$	$(11)(2) = 22$
$(8)(4) = 32$	$3 \cdot 5 \cdot 5 = 75$	$8(9) = 72$
$4(4) = 16$	$3(9) = 27$	$10 \cdot 10 = 100$
$9 \cdot 10 = 90$	$1 \cdot 5 = 5$	$6 \cdot 5 = 30$
$6 \cdot 6 = 36$	$4 \cdot 9 = 36$	$9 \cdot 9 = 81$

Below is a multiplication table that needs to be finished. Finish it and then use it to check the problems you just did.

	1	2	3	4	5	6	7	8	9	10	11	12
1	**1**	**2**	**3**	**4**	**5**	**6**	7	8	9	10	11	12
2	**2**	**4**	**6**	**8**	10	12	14	16	18	20	22	24
3	**3**	**6**	**9**	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	**48**	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Factoring

In a multiplication problem like

$$5 \cdot 7 = 35$$

5 and 7 are called **factors** of 35. Factors are numbers which are multiplied. Many times in algebra we have to break down a number into factors. Both factors will have to be whole numbers. Here are some examples:

~~24~~ 12 · 2 $\boxed{24 = 12 \cdot 2}$	~~14~~ 2 · 7 $\boxed{14 = 2 \cdot 7}$	~~64~~ 8 · 8 $\boxed{64 = 8 \cdot 8}$

Below are some numbers for you to factor. See if you can factor each number without using any fractions. Many have more than one answer

~~15~~ 3 · 5 $\boxed{15 = 3 \cdot 5}$	27 3 9 $\boxed{27 = 3 \cdot 9}$	8 2 4 $\boxed{8 = 2 \cdot 4}$
16 2 8 $\boxed{16 = 2 \cdot 8}$	50 5 10 $\boxed{50 = 5 \cdot 10}$	72 9 8 $\boxed{72 = 9 \cdot 8}$
20 5 4 $\boxed{20 = 5 \cdot 4}$	36 6 6 $\boxed{36 = 6 \cdot 6}$	22 2 11 $\boxed{22 = 2 \cdot 11}$

63 3 21 $63 = 3 \cdot 21$	56 7 8 $56 = 7 \cdot 8$	28 2 14 $28 = 2 \cdot 14$
81 9 9 $81 = 9 \cdot 9$	49 7 7 $49 = 7 \cdot 7$	100 10 10 $100 = 10 \cdot 10$
77 7 11 $77 = 7 \cdot 11$	39 3 13 $39 = 3 \cdot 13$	132 3 44 $132 = 3 \cdot 44$
30 3 10 $30 = 3 \cdot 10$	30 2 15 $30 = 2 \cdot 15$	30 5 6 $30 = 5 \cdot 6$
48 2 24 $48 = 2 \cdot 24$	48 4 12 $48 = 4 \cdot 12$	48 12 4 $48 = 12 \cdot 4$
17 1 17 $17 = 1 \cdot 17$	5 5 1 $5 = 5 \cdot 1$	23 1 23 $23 = 1 \cdot 23$

Prime Numbers

Let's try to factor the number 17. The only way we can do it is like this:

$$\cancel{17}$$

$$1 \cdot 17$$

$$\boxed{17 = 1 \cdot 17}$$

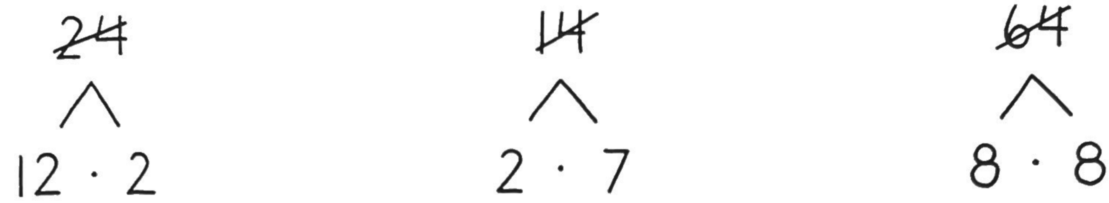
We still end up with a 17, so you can see that we really didn't break down the 17 into two smaller numbers. In fact, that's impossible to do without using fractions. A number, like 17, that can only be factored into 1 times itself is called a **prime number**. 5 is also a prime number, since 1 • 5 is the only way it can be factored using whole number Can you think of some more prime numbers? (Note: 1 is not considered a prime number. 2 is the smallest prime number.)

Try to factor each number into a product of smaller numbers. If a number is prime, draw a circle around it.

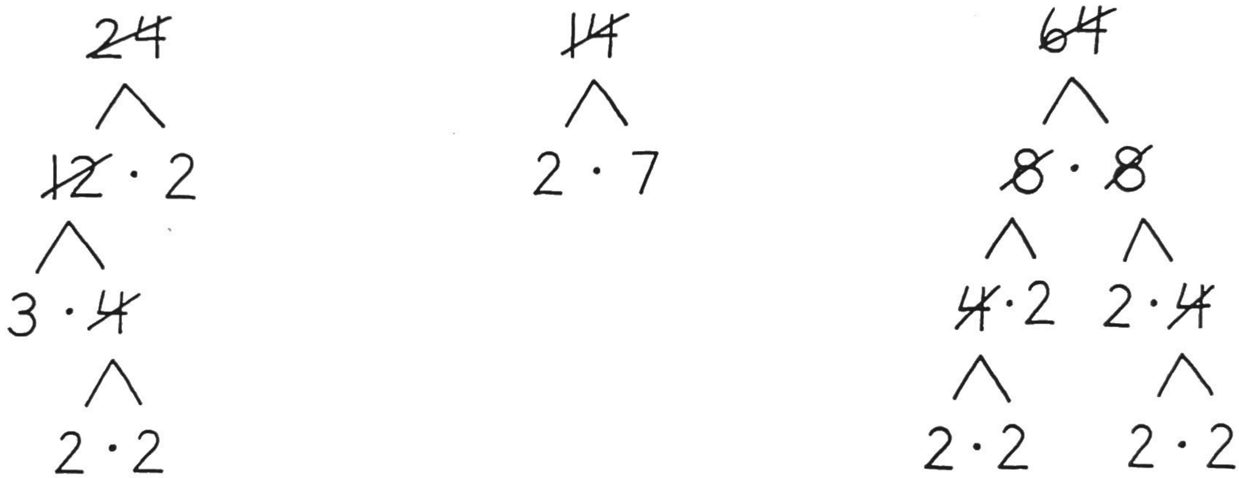
2	3	~~4~~	5	~~6~~	7	8
prime	prime	2 • 2	prime	2 • 3	prime	2 4
9	10	11	12	13	14	15
3 3	5 2	prime	2 6	prime	2 7	5 3
16	17	18	19	20	21	22
4 4	prime	2 9	prime	2 10	7 3	2 11

Prime Factors

Here are some numbers that we have already broken down:

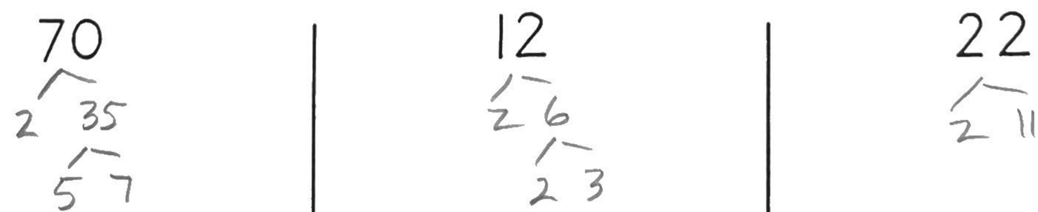


However, only one of them has been broken down all the way. This time we are going to break them all the way down into **prime factors**:



24 = 3·2·2·2	14 = 2 · 7	64 = 2·2·2·2·2·2

Break down each number into prime factors.



70 = 2·5·7	12 = 2·2·3	22 = 2·11

~~20~~

~~4~~ · 5

2 · 2

$\boxed{20 = 2 \cdot 2 \cdot 5}$

90

2 45

5 9

3 3

$\boxed{90 = 2 \cdot 5 \cdot 3 \cdot 3}$

24

2 12

2 6

2 3

$\boxed{24 = 2 \cdot 2 \cdot 2 \cdot 3}$

18

2 9

3 3

$\boxed{18 = 2 \cdot 3 \cdot 3}$

81

3 27

3 9

3 3

$\boxed{81 = 3 \cdot 3 \cdot 3 \cdot 3}$

30

2 15

3 5

$\boxed{30 = 2 \cdot 3 \cdot 5}$

48

2 24

2 12

2 6

2 3

$\boxed{48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$

100

2 50

2 25

5 5

$\boxed{100 = 2 \cdot 2 \cdot 5 \cdot 5}$

32

2 16

2 8

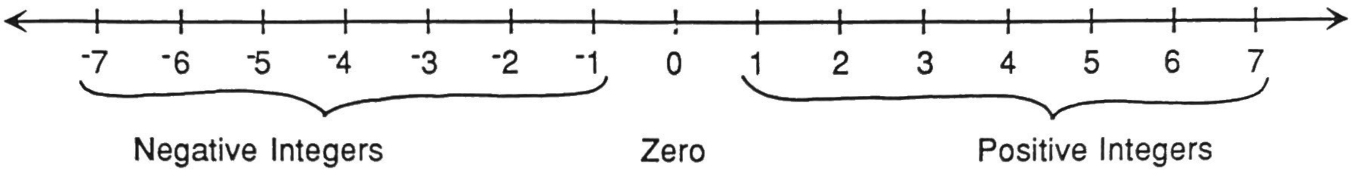
2 4

2 2

$\boxed{32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

Integers

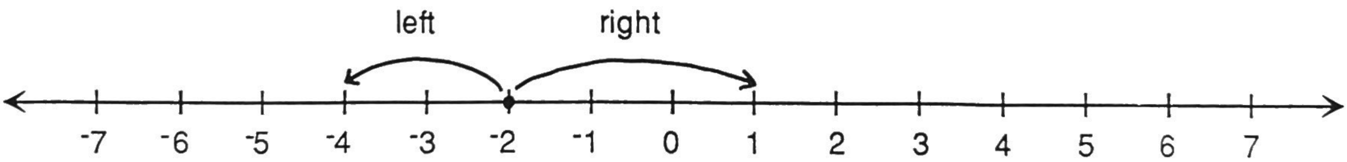
Integers are a lot like the whole numbers that you already know. The main difference is that there are **negative integers** as well as **positive integers**. **Zero** is also an integer. Here is one way we can picture the set of integers:



As you can see, the negative integers are to the left of zero. We use a little raised minus sign to show that an integer is negative. Sometimes we use a little raised plus sign to show that an integer is positive, but we usually don't use any sign at all when the integer is positive. Zero is neither positive nor negative.

Comparing Integers

By looking at the number line we can easily tell which integers are **greater** (larger) than a certain number and which are **less** (smaller) than the number.



1 is greater than ⁻2 because it is to the *right* of ⁻2.

⁻4 is less than ⁻2 because it is to the *left* of ⁻2.

Write >, = or < between the two integers to show whether the first is greater than, equal to or less than the second.

1 is greater than ⁻2.

⁻4 is less than ⁻2.

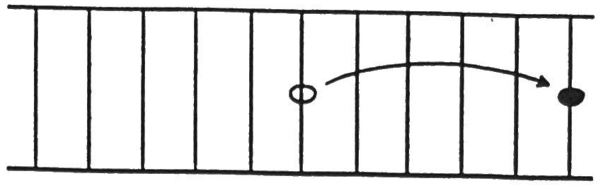
1 > ⁻2	⁻4 < ⁻2	⁻2 > ⁻4
5 > 2	⁻7 < ⁻3	2 > ⁻4
2 < 5	⁻4 = ⁻4	⁻5 < 5
⁻2 < 0	3 > ⁻3	1 > ⁻1
0 < 2	⁻2 > ⁻6	⁻1 < 0

Showing Gains and Losses

You can also use integers to show **gains** and **losses**. Positive integers show gains, and negative integers show losses. Zero is used if there is no change. Here is how we can use integers to show gains and losses in a football game:

We gained 5 yards.

5

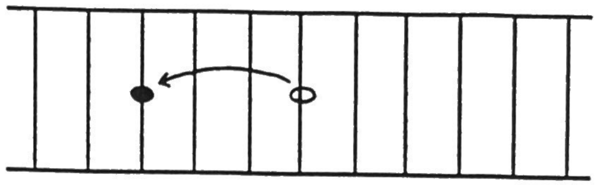


positive 5

5

We lost 3 yards.

⁻3

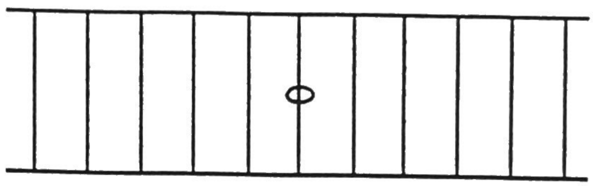


negative 3

⁻3

There was no change (no gain and no loss).

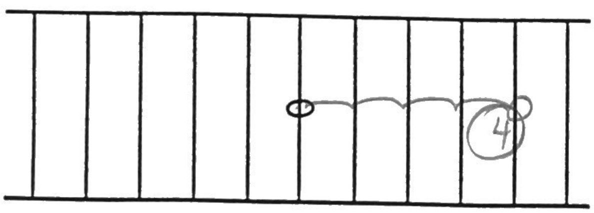
0



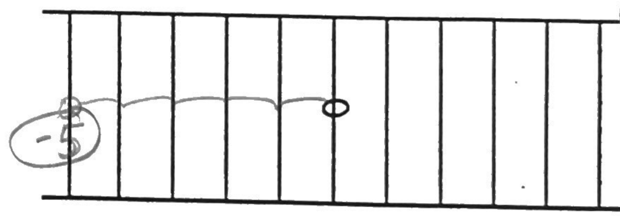
zero (not positive and not negative)

0

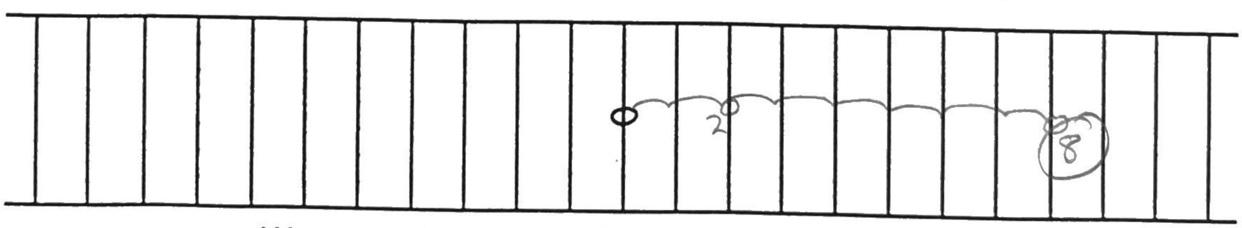
You show the following gains and losses.



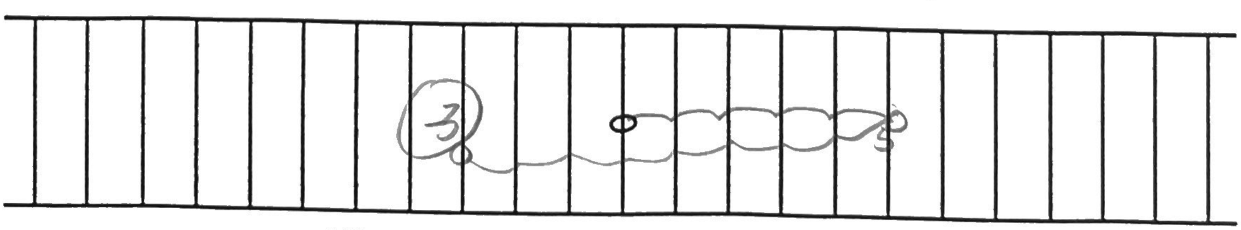
We gained 4 yards.



We lost 5 yards.



We gained 2 yards and then gained 6 more yards.

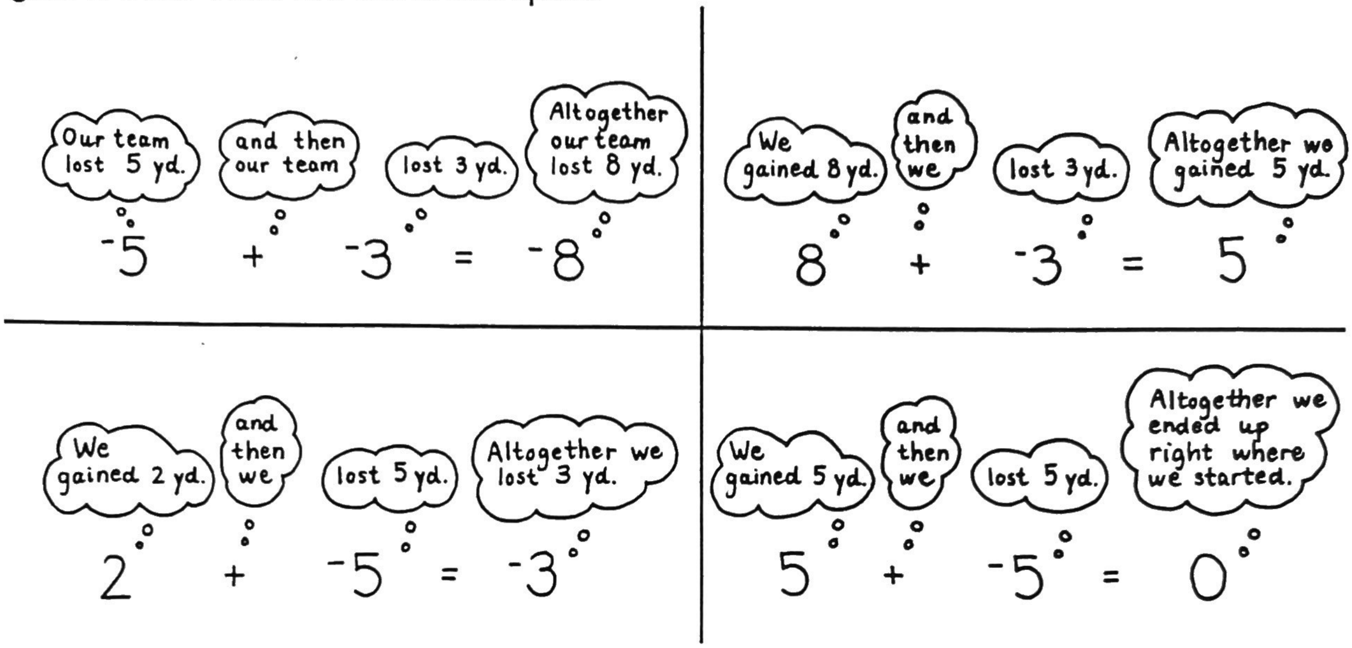


We gained 5 yards and then lost 8 yards.

Adding Integers

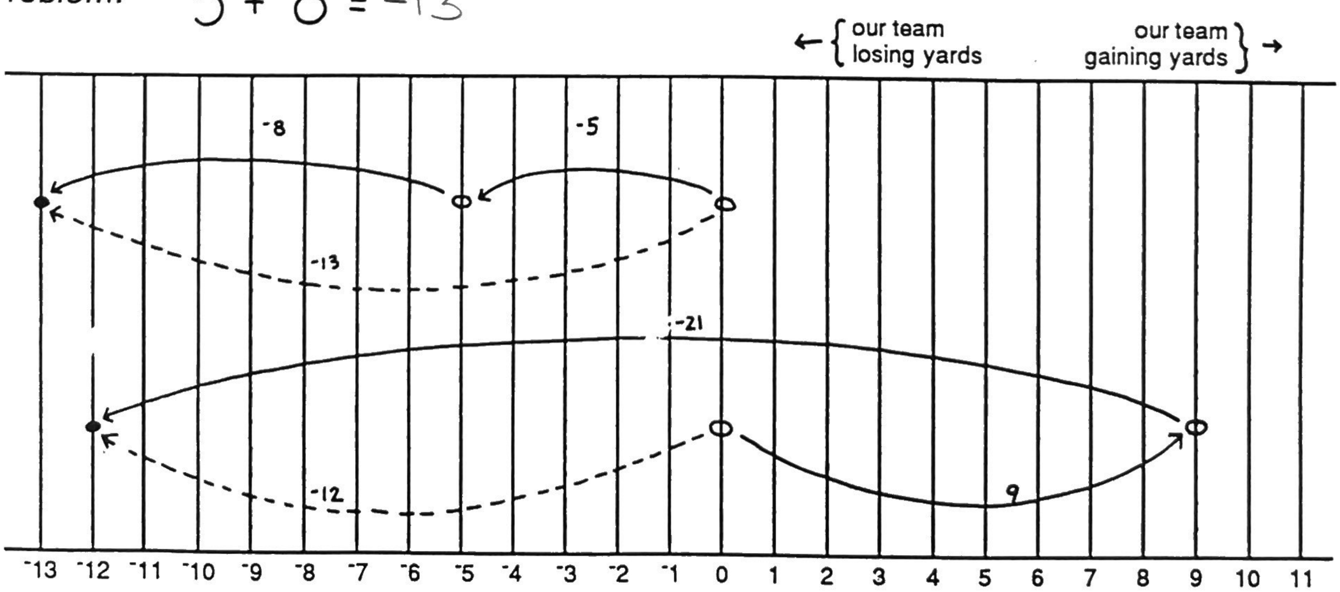
In arithmetic you learned the operations of adding, subtracting, multiplying and dividing whole numbers and fractions. In algebra, one of the first things you have to learn is how to add, subtract, multiply and divide integers.

To add integers we can think of a football game. A positive number stands for ground gained by our team; a negative number shows ground lost. Zero is used when there is no gain or loss. Here are some examples:

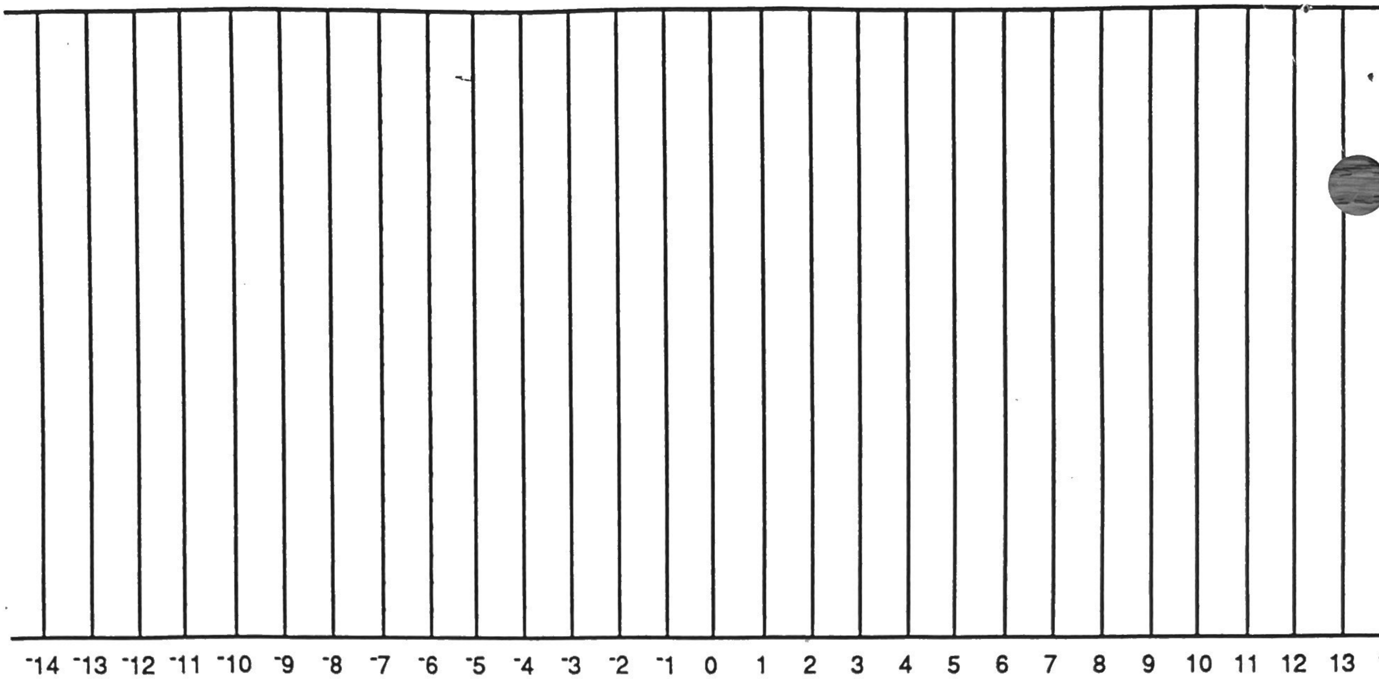


If you ever have trouble adding integers, then you can draw a football field to help you figure out the answer.

Problem: $^{-}5 + {}^{-}8 = -13$



Problem: $9 + {}^{-}21 = -12$



Use the football field to help you do each problem below.

$^-3 + {}^-5 = -8$

$6 + 4 = 10$

$8 + {}^-2 = 6$

$7 + {}^-7 = 0$

$^-3 + 5 = 2$

$^-4 + 9 = 5$

$6 + {}^-13 = -7$

$^-5 + 5 = 0$

$^-6 + {}^-6 = -12$

$4 + 2 = 6$

$5 + {}^-3 = 2$

$13 + {}^-4 = 9$

$8 + {}^-9 = -1$

$^-5 + {}^-6 = -11$

$^-14 + 6 = -8$

$^-1 + 10 = 9$

$1 + {}^-10 = -9$

$^-12 + 0 = -12$

$13 + {}^-13 = 0$

$10 + {}^-20 = -10$

$8 + {}^-16 = -8$

$^-12 + 25 = 13$

$^-14$ $^-13$ $^-12$ $^-11$ $^-10$ $^-9$ $^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Add.

$^-6 + ^-3 = -9$	$4 + 5 = 9$	$3 + 2 = 5$	$7 + 6 = 13$
$8 + ^-5 = 3$	$^-9 + ^-2 = -11$	$3 + 1 = 4$	$8 + ^-4 = 4$
$^-5 + ^-3 = -8$	$^-5 + ^-8 = -13$	$3 + 0 = 3$	$9 + ^-7 = 2$
$^-8 + 8 = 0$	$^-9 + 4 = -5$	$3 + ^-1 = 2$	$^-7 + ^-7 = -14$
$^-6 + 5 = -1$	$10 + ^-8 = 2$	$3 + ^-2 = 1$	$^-7 + 9 = 2$
$12 + ^-6 = 6$	$2 + ^-11 = -9$	$3 + ^-3 = 0$	$^-11 + 5 = -6$
$^-11 + 0 = -11$	$13 + ^-9 = 4$	$3 + ^-4 = -1$	$^-1 + 1 = 0$
$^-16 + 7 = -9$	$^-13 + 5 = -8$	$3 + ^-5 = -2$	$17 + ^-8 = 9$
$8 + ^-3 = 5$	$^-5 + 9 = 4$	$3 + ^-6 = -3$	$^-14 + 14 = 0$

$$\begin{array}{r} 2 \\ +\ 6 \\ \hline 8 \end{array} \quad \begin{array}{r} ^-5 \\ +\ ^-6 \\ \hline -11 \end{array} \quad \begin{array}{r} 6 \\ +\ ^-9 \\ \hline -3 \end{array} \quad \begin{array}{r} ^-7 \\ +\ ^-4 \\ \hline -11 \end{array} \quad \begin{array}{r} 9 \\ +\ ^-3 \\ \hline 6 \end{array} \quad \begin{array}{r} ^-11 \\ +\ 4 \\ \hline -7 \end{array} \quad \begin{array}{r} ^-12 \\ +\ 15 \\ \hline 3 \end{array} \quad \begin{array}{r} ^-11 \\ +\ 11 \\ \hline 0 \end{array}$$

$14 + 0 = 14$	$^-6 + 0 = -6$	$37 + 0 = 37$	$^-65 + 0 = -65$
$0 + 3 = 3$	$0 + ^-8 = -8$	$0 + ^-15 = -15$	$0 + 39 = 39$

Adding zero is easy! We just have to look at the other number in the problem and that's the answer. Here is a way we can say this:

> If a is any integer, then $a + 0 = a$
> and $0 + a = a$.

This is called the Principle for Adding Zero.

$^{-}3 + 7 = 4$ $7 + {}^{-}3 = 4$	$5 + 8 = 13$ $8 + 5 = 13$	$^{-}6 + {}^{-}2 = -8$ $^{-}2 + {}^{-}6 = -8$
$^{-}16 + {}^{-}5 = -21$ $^{-}5 + {}^{-}16 = -21$	$^{-}16 + 18 = 2$ $18 + {}^{-}16 = 2$	$^{-}47 + 47 = 0$ $47 + {}^{-}47 = 0$
$^{-}3 + 35 = 32$ $35 + {}^{-}3 = 32$	$30 + {}^{-}80 = -50$ $^{-}80 + 30 = -50$	$^{-}5 + {}^{-}21 = -26$ $^{-}21 + {}^{-}5 = -26$
$^{-}100 + {}^{-}62 = -162$ $^{-}62 + {}^{-}100 = -162$	$100 + 99 = 199$ $99 + 100 = 199$	$100 + {}^{-}25 = 75$ $^{-}25 + 100 = 75$
$100 + {}^{-}99 = 1$ $^{-}99 + 100 = 1$	$100 + {}^{-}53 = 47$ $^{-}53 + 100 = 47$	$100 + {}^{-}77 = 23$ $^{-}77 + 100 = 23$

As you can see from the problems on this page, it doesn't matter which of the numbers comes first when we are adding. We get the same answer either way. This is what we mean when we say that addition of integers is **commutative**.

> Commutative Principle for Addition of Integers:
> If a and b are integers, then $a + b = b + a$.

For example, $\underbrace{^{-}3 + 7}_{4} = \underbrace{7 + {}^{-}3}_{4}$

On this page, the parentheses tell you which pair of numbers to add first.

$$\overbrace{(5+{}^{-}3)}^{2}+{}^{-}6 = -4$$
$$5+\underbrace{({}^{-}3+{}^{-}6)}_{-9} = -4$$

$$\overbrace{({}^{-}4+{}^{-}8)}^{-12}+{}^{-}7 = -19$$
$${}^{-}4+\underbrace{({}^{-}8+{}^{-}7)}_{-15} = -19$$

$$\overbrace{(6+{}^{-}4)}^{2}+7 = 9$$
$$6+\underbrace{({}^{-}4+7)}_{3} = 9$$

$$\overbrace{({}^{-}3+7)}^{4}+{}^{-}5 = -1$$
$${}^{-}3+\underbrace{(7+{}^{-}5)}_{2} = -1$$

$$\overset{-1}{({}^{-}8+7)}+5 = 4$$
$${}^{-}8+\underset{12}{(7+5)} = 4$$

$$\overset{-13}{({}^{-}3+{}^{-}10)}+6 = -7$$
$${}^{-}3+\underset{-4}{({}^{-}10+6)} = -7$$

$$\overset{-15}{({}^{-}6+{}^{-}9)}+9 = -6$$
$${}^{-}6+\underset{0}{({}^{-}9+9)} = -6$$

$$\overset{-100}{({}^{-}23+{}^{-}77)}+{}^{-}57 = -157$$
$${}^{-}23+\underset{-134}{({}^{-}77+{}^{-}57)} = -157$$

These problems show that it makes no difference which pair of numbers we add first. The answer always comes out the same. This is what we mean when we say that addition of integers is **associative.**

> Associative Principle for Addition of Integers:
> If a, b and c are integers, then $(a+b)+c = a+(b+c)$.

For example, $\underbrace{({}^{-}8+7)+5}_{4} = \underbrace{{}^{-}8+(7+5)}_{4}$

Write a positive or negative number or zero for each sentence.

The Bears lost 16 yards.	-16
The Raiders gained 38 yards.	38
Don won 80¢.	.80
Harry lost 65¢.	-.65
Tom broke even.	0
The temperature went up 8 degrees.	8
The temperature went down 13 degrees.	-13
Marty lost 5 kilograms.	-5
Irene gained 3 kilograms.	3
Barbara stayed the same weight.	0
Mr. Green spent $20.	-20
Mrs. Williams earned $86.	86
Brenda lost $3.	-3
Theresa found $1.	1
Carla didn't find anything.	0
The water level fell 7 centimeters.	-7

Write a problem for each sentence.

The Giants gained 8 yards and then lost 5 yards.	8 + -5
The temperature fell 6 degrees and then fell 5 more degrees.	-6 + -5
Marty lost 5 kilograms but then gained back 3.	-5 + 3
Mrs. Williams got paid $86 and then spent $40.	86 + -40
Mr. Lopez lost $3, but then he found $2 of it.	-3 + 2
The water level rose 11 cm and then fell 11 cm.	11 + -11
An airplane climbed 2000 meters, then climbed another 1500 meters.	2000 + 1500

For each exercise below you have to do two things.

1. First write down the problem.
2. Then find the answer.

The Jackets football team gained 9 yards on their first play. On the next three plays they lost 4 yards, gained 3 yards and lost 3 yards. How did the team do altogether on these four plays?

Problem: $9 + {}^{-}4 + 3 + {}^{-}3 = 5$

Answer: They gained 5 yards.

The Raiders gained 2 yards, lost 18 yards, lost 2 yards and then gained 10 yards. How did they do on these four plays?

Problem: $2 + -18 + -2 + 10$

Answer: -8

James was playing a game with his friends. He won 35 points. Then he lost 15, lost 40 and won 55. How did he come out?

Problem: $35 + -15 + 40 + 55$

Answer: 115

Donna won 43 points, lost 17, lost 19, won 17, lost 24, won 19 and lost 43. How did she come out?

Problem: $43 + -17 + -19 + 17 + -24 + 19 + -43$

Answer: -24

Shirlee had a savings account. Her first deposit was \$35. Then she deposited \$10, withdrew \$20, withdrew \$5, deposited \$50, withdrew \$10 and deposited \$25. How much does she now have in her account?

Problem: $35 + 10 + -20 + -5 + 50 + -10 + 25$

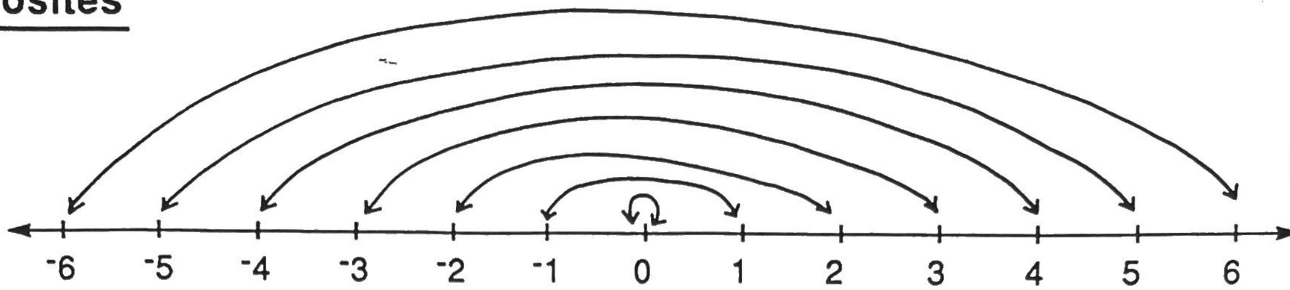
Answer: 85

Mr. Jackson had \$100 in his checking account. He wrote checks for \$30, \$15 and \$20. Then he deposited \$50 and wrote checks for \$75, \$15 and \$10. How much does Mr. Jackson have in his account now? (Do you see why the bank doesn't want Mr. Jackson to write any more checks?)

Problem: $100 + -30 + -15 + -20 + 50 + -75 + -15 + -10$

Answer:

Opposites



See how the integers are matched up in the picture? Each integer is matched with its **opposite**. We use a dash to show the opposite of a number.

The opposite of 5 is $^-5$.	$-(5) = {}^-\mathbf{5}$
The opposite of $^-5$ is 5.	$-(^-5) = \mathbf{5}$
The opposite of 0 is 0.	$-(0) = \mathbf{0}$
The opposite of $^-13$ is 13.	$-(^-13) = 13$
The opposite of 13 is $^-13$.	$-(13) = -13$
The opposite of $^-8$ is 8.	$-(^-8) = 8$
The opposite of 8 is $^-8$.	$-(8) = -8$

Here are some problems where you have to add opposites:

$5 + {}^-5 = 0$ $\qquad$ $^-5 + 5 = 0$

$^-3 + 3 = 0$ $\qquad$ $3 + {}^-3 = 0$

$46 + {}^-46 = 0$ $\qquad$ $^-8 + 8 = 0$

$0 + 0 = 0$ $\qquad$ $8 + {}^-8 = 0$

$167 + {}^-167 = 0$ $\qquad$ $^-3647 + 3647 = 0$

As you can see, whenever we add two opposites they cancel each other out — the answer always comes out zero.

If a is an integer, then $\quad a + -a = 0$ and $\quad -a + a = 0$.

This is called the Principle for Adding Opposites.

The problems on this page are too hard . . .

Make them easier by finding opposites and getting rid of them.

$8 + {}^{-}8 = 0$

$\cancel{8} + 6 + \cancel{{}^{-}8} + 4 = 10$

$\cancel{5} + 2 + \cancel{{}^{-}5} + 6 = 8$

${}^{-}7 + \cancel{6} + {}^{-}4 + \cancel{{}^{-}6} = -11$

$\cancel{9} + {}^{-}2 + 8 + \cancel{{}^{-}9} = 6$

$\cancel{3} + \cancel{{}^{-}8} + \cancel{{}^{-}3} + \cancel{8} = 0$

$\cancel{37} + 4 + \cancel{{}^{-}37} + {}^{-}5 = -1$

$\cancel{6} + \cancel{{}^{-}9} + 8 + \cancel{9} + \cancel{{}^{-}6} + 3 = 11$

$\cancel{{}^{-}8} + \cancel{8} + \cancel{5} + \cancel{9} + \cancel{{}^{-}5} + 6 + \cancel{{}^{-}9} = 6$

$\cancel{17} + \cancel{{}^{-}28} + 56 + \cancel{28} + \cancel{{}^{-}17} = 56$

$\cancel{{}^{-}12} + \cancel{5} + {}^{-}6 + \cancel{12} + \cancel{{}^{-}5} = -6$

James is still playing with his friends. He won 25 points. Then he lost 17, won 2, lost 19, won 2, won 17, lost 33, won 19, lost 25 and won 33. How did he come out?

Problem: $25 + -17 + 2 + -19 + 2 + 17 + -33 + 19 + -25 + 33$

Answer: 4

Subtracting Integers

Once we have learned how to add and find opposites of integers, it is easy to subtract them. Every subtraction problem has the same answer as an addition problem.

$$\begin{cases} 8 - 5 = 3 \\ 8 + {}^{-}5 = 3 \end{cases} \qquad \begin{cases} 10 - 1 = 9 \\ 10 + {}^{-}1 = 9 \end{cases}$$

To find the answer to a subtraction problem, all we have to do is change it to an addition problem — but instead of subtracting the second number we *add the opposite of the second number.*

Here is another example: $^{-}5 - 4 =$

Instead of subtracting positive 4 we are going to add negative 4; so here's how you can change the problem:

$$^{-}5 + {}^{-}4 =$$

Now the problem is just like the adding problems we have already done.
A loss of 5 and a loss of 4 comes out to a loss of 9:

$$^{-}5 + {}^{-}4 = {}^{-}9$$

Below are some more subtraction problems. Change each problem to an addition problem. Remember to add the *opposite* of the *second number.*

$6 + {}^{-}8 = {}^{-}2$ | $4 + {}^{-}9 = -5$

$^{-}4 + {}^{-}8 = -12$ | $^{-}5 + {}^{-}3 = -8$

$5 + {}^{-}9 = -4$ | $^{-}9 + {}^{-}4 = -13$

$8 + {}^{-}5 = 3$ | $4 + {}^{-}7 = -3$

$^{-}3 + {}^{-}6 = -9$ | $7 + {}^{-}2 = 5$

$^{-}8 + {}^{-}8 = -16$ | $^{-}1 + {}^{-}6 = -7$

In this subtraction problem the number being subtracted is negative:

$$7 - {}^{-}5 =$$

First we have to change the problem. We have to add the opposite of the second number, so instead of subtracting negative 5 we are going to add positive 5:

$$7 + {}^{+}5 =$$

A gain of 7 and a gain of 5 is the same as a gain of 12:

$$7 + {}^{+}5 = 12$$

If this seems strange to you, think of the football field. When the referee takes away or rules out a 5 yard loss, we gain back the 5 yards.

Subtract.

${}^{-}3 + {}^{+}7 = 4$

$5 + {}^{+}9 = 14$

$8 + {}^{+}3 = 11$

${}^{-}6 + {}^{+}4 = -2$

${}^{-}9 + {}^{+}6 = -3$

${}^{-}8 + {}^{+}8 = 0$

$10 + {}^{+}5 = 15$

${}^{-}10 + {}^{+}5 = -5$

$2 + {}^{+}9 = 11$

${}^{-}2 + {}^{+}9 = 7$

Be careful on these.

$6 + {}^{+}8 = 14$

$6 + {}^{-}8 = -2$

${}^{-}6 + {}^{+}8 = 2$

${}^{-}6 + {}^{-}8 = -14$

As you can see, every time we have a subtraction problem, we can change it to an adding problem. But we have to remember to *add the opposite* of the second number:

$$\boxed{\text{If } a \text{ and } b \text{ are integers, then } a-b = a+-b.}$$

Subtract. Remember to add the *opposite* of the second number.
(If the second number is positive, change it to negative.
If the second number is negative, change it to positive.)

$4 + {}^{-}2 = 2$	$5 + {}^{+}3 = 8$	${}^{-}3 + {}^{+}4 = 1$	${}^{-}6 + {}^{-}2 = {}^{-}8$
${}^{-}6 + {}^{+}3 = -3$	$4 + {}^{-}5 = -1$	${}^{-}3 + {}^{+}7 = 4$	$7 + {}^{-}6 = 1$
$8 + {}^{+}5 = 13$	${}^{-}9 + {}^{+}2 = -7$	${}^{-}2 + {}^{-}7 = -9$	$8 + {}^{+}4 = 12$
${}^{-}5 + {}^{+}3 = -2$	$5 + {}^{-}9 = -4$	$2 + {}^{+}7 = 9$	$9 + {}^{+}7 = 16$
${}^{-}8 + {}^{-}8 = -16$	${}^{-}5 + {}^{+}8 = 3$	$1 + -9 = -8$	${}^{-}7 + {}^{+}7 = 0$
$8 + {}^{+}3 = 11$	${}^{-}9 + {}^{-}4 = -13$	${}^{-}8 + {}^{+}3 = -5$	$7 + -9 = -2$
${}^{-}6 + {}^{+}5 = -1$	$10 + {}^{+}8 = 18$	${}^{-}12 + {}^{+}4 = -8$	${}^{-}11 + {}^{-}5 = -16$
$12 + {}^{+}6 = 18$	$11 + {}^{+}2 = 12$	$0 + {}^{-}5 = -5$	${}^{-}1 + -1 = -2$
${}^{-}11 + {}^{-}0 = -11$	$13 + {}^{+}9 = 22$	${}^{-}15 + {}^{+}6 = -9$	$17 + {}^{+}8 = 25$
${}^{-}16 + -7 = -23$	${}^{-}13 + {}^{-}5 = -18$	$18 + {}^{+}9 = 27$	${}^{-}14 + {}^{-}14 = -28$
$14 + {}^{+}5 = 19$	$8 + {}^{-}9 = -1$	$4 + {}^{-}8 = -4$	$0 + {}^{+}6 = 6$
${}^{-}10 + {}^{-}10 = -20$	${}^{-}1 + {}^{+}8 = 7$	$7 + -7 = 0$	${}^{-}14 + {}^{+}4 = -10$

Subtract.
(Remember . . .
add the opposite of
the bottom number.)

$\begin{array}{r} {}^{-}5 \\ + \; {}^{+}6 \\ \hline 1 \end{array}$	$\begin{array}{r} 6 \\ + \; {}^{+}9 \\ \hline 15 \end{array}$	$\begin{array}{r} 2 \\ + \; -6 \\ \hline -4 \end{array}$	$\begin{array}{r} {}^{-}8 \\ + \; {}^{+}4 \\ \hline -4 \end{array}$

$\begin{array}{r} 6 \\ + \; {}^{+}9 \\ \hline 15 \end{array}$	$\begin{array}{r} {}^{-}11 \\ + \; -4 \\ \hline -15 \end{array}$	$\begin{array}{r} 14 \\ + \; {}^{+}8 \\ \hline 22 \end{array}$	$\begin{array}{r} 16 \\ + \; -13 \\ \hline 3 \end{array}$	$\begin{array}{r} {}^{-}16 \\ + \; {}^{+}13 \\ \hline -3 \end{array}$	$\begin{array}{r} {}^{-}10 \\ + \; -10 \\ \hline -20 \end{array}$

Add or subtract as indicated.

$^{-}6 + 10 = 4$ $^{-}6 + ^{-}10 = -16$	$^{-}3 + ^{-}11 = -14$ $^{-}3 + ^{+}11 = 8$
$9 + 7 = 16$ $9 + ^{-}7 = -2$	$6 + ^{-}9 = -3$ $6 + ^{+}9 = 15$
$^{-}8 + 2 = -6$ $^{-}8 + ^{-}2 = -10$	$^{-}7 + ^{-}6 = -13$ $^{-}7 + ^{+}6 = -1$
$3 + 10 = 13$ $3 + ^{-}10 = -7$	$12 + ^{-}8 = 4$ $12 + ^{+}8 = 20$
$^{-}35 + ^{-}15 = -50$ $^{-}35 + ^{+}15 = -20$	$^{-}18 + 18 = 0$ $^{-}18 + ^{-}18 = -36$

Add. (Think of gaining and losing yards.)

$^{-}9$	$^{-}5$	10	6	$^{-}15$	$^{-}8$
$+\ ^{-}4$	$+\ 8$	$+\ ^{-}7$	$+\ 7$	$+\ 5$	$+\ 8$
-13	3	3	13	-10	0

$^{-}37$	$^{-}53$	25	$^{-}84$	$^{-}43$	$^{-}25$
$+\ 26$	$+\ ^{-}42$	$+\ ^{-}26$	$+\ 86$	$+\ 0$	$+\ ^{-}25$
-11	-95	-1	2	-43	-50

Subtract. (Add the opposite of the bottom number.)

$^{-}9$	$^{-}5$	10	6	$^{-}15$	$^{-}8$
$+\ ^{+}4$	$+\ -8$	$+\ ^{+}7$	$+\ -7$	$+\ -5$	$+\ -8$
-5	-13	17	-1	-20	-16

$^{-}37$	$^{-}53$	25	$^{-}84$	$^{-}43$	$^{-}25$
$+\ ^{-}26$	$+\ ^{+}42$	$+\ ^{+}26$	$+\ ^{-}86$	$+\ -0$	$+\ ^{+}25$
-63	-11	51	-170	-43	0

Here are some longer problems.

$13 + {}^{-}3 + {}^{-}6 + {}^{-}1 = 3$

$16 + {}^{-}10 + {}^{-}2 + {}^{-}3 = 1$

$8 + {}^{-}2 + {}^{-}2 + {}^{-}2 = 2$

$10 + {}^{-}3 + {}^{-}3 + {}^{-}4 = 0$

$9 + {}^{-}3 + {}^{-}6 + {}^{-}5 = -5$

$12 + {}^{-}7 + {}^{-}5 + {}^{-}4 = -4$

$8 + {}^{-}5 + {}^{-}5 + {}^{-}5 = -7$

$6 + {}^{-}5 + {}^{-}4 + {}^{-}3 = -6$

$11 + {}^{-}9 + {}^{-}6 + {}^{-}6 = -10$

$3 + {}^{-}7 + {}^{-}4 + {}^{-}2 = -10$

$12 + {}^{-}15 + {}^{-}10 + {}^{-}8 = -21$

$7 + {}^{-}7 + {}^{-}7 + {}^{-}7 = -14$

Multiplying Integers

To name an integer we have to do two things. We have to tell the *sign* of the integer (positive or negative), and we also have to tell the *amount*.

When we multiply two integers, we need to break down the problem into two parts. First we figure out the sign of the answer; then we figure out the amount of the answer. To figure out the *sign* of the answer, all we have to do is remember these four rules:

POSITIVE • POSITIVE = POSITIVE
POSITIVE • NEGATIVE = NEGATIVE
NEGATIVE • POSITIVE = NEGATIVE
NEGATIVE • NEGATIVE = POSITIVE

To figure out the *amount* of the answer, we just multiply.

Multiply.

$8 \cdot {}^-3 = -24$

${}^-8 \cdot {}^-3 = 24$

$8 \cdot 3 = 24$

${}^-8 \cdot 3 = -24$

${}^-6 \cdot {}^-6 = 36$

$6 \cdot {}^-6 = -36$

${}^-6 \cdot 6 = -36$

$6 \cdot 6 = 36$

$2 \cdot 0 = 0$

${}^-2 \cdot 0 = 0$

$2 \cdot {}^-2 = -4$

$5 \cdot {}^-4 = -20$

${}^-8 \cdot 2 = -16$

$5 \cdot {}^-6 = -30$

${}^-3 \cdot {}^-7 = 21$

${}^-8 \cdot {}^-1 = 8$

$4 \cdot 8 = 32$

${}^-9 \cdot {}^-8 = 72$

${}^-7 \cdot 7 = -49$

$1 \cdot {}^-3 = -3$

$5 \cdot 0 = 0$

${}^-4 \cdot {}^-7 = 28$

$0 \cdot {}^-9 = 0$

$0 \cdot 8 = 0$

${}^-5 \cdot 4 = -20$

${}^-6 \cdot 7 = -42$

$8 \cdot 5 = 40$

${}^-4 \cdot {}^-6 = 24$

$9 \cdot {}^-7 = -63$

$7 \cdot {}^-9 = -63$

${}^-3 \cdot 6 = -18$

${}^-5 \cdot {}^-7 = 35$

${}^-1 \cdot 9 = -9$

${}^-10 \cdot {}^-8 = 80$

$9 \cdot {}^-10 = -90$

$8 \cdot 7 = 56$

${}^-11 \cdot 3 = -33$

To see why these rules work we can think again about a football game. The *first integer* tells how many plays were made. A positive integer tells how many plays were made by *our* team; a negative integer tells how many plays were made by the *other* team. The *second integer* tells how many yards were gained or lost on each play. Positive integers show yards gained; negative integers show yards lost. The *answer* tells the outcome in terms of how many yards *our* team gained or lost. Here are some examples:

Our team made 3 plays and gained 5 yd. on each play. Altogether we gained 15 yd.

$$3 \cdot 5 = 15$$

Our team made 3 plays and lost 5 yd. on each play. Altogether we lost 15 yd.

$$3 \cdot {}^{-}5 = {}^{-}15$$

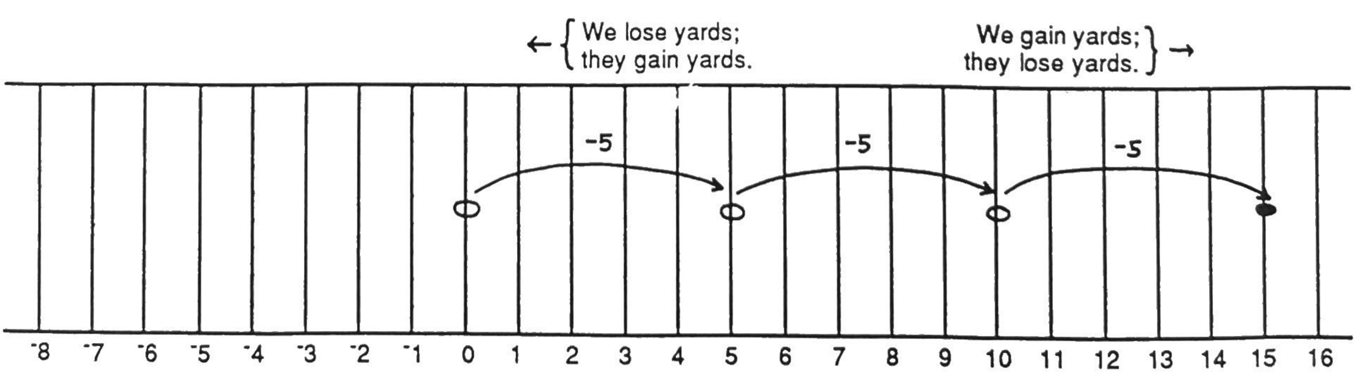
The **other** team made 3 plays and gained 5 yd. on each play. Altogether **we** lost 15 yd.

$$^{-}3 \cdot 5 = {}^{-}15$$

The **other** team made 3 plays and lost 5 yd. on each play. Altogether **we** gained 15 yd.

$$^{-}3 \cdot {}^{-}5 = 15$$

If it seems strange that we get a positive answer by multiplying two negative numbers, just remember that a loss for the other team is a gain for us. We can draw a football field and map out the plays to see why this is so.



Problem: $^{-}3 \cdot {}^{-}5 = 15$

Multiply.

$6 \cdot 7 = 42$

$^{-}6 \cdot {}^{-}7 = 42$

$6 \cdot {}^{-}7 = -42$

$^{-}6 \cdot 7 = -42$

Here's an easy way to tell the sign of the answer:

Both positive or both negative	$4 \cdot 6 = 24$ $^{-}4 \cdot {}^{-}6 = 24$	Answer is positive.
One positive and one negative	$4 \cdot {}^{-}6 = {}^{-}24$ $^{-}4 \cdot 6 = {}^{-}24$	Answer is negative.

$^{-}9 \cdot {}^{-}5 = 45$

$9 \cdot {}^{-}5 = -45$	$^{-}8 \cdot {}^{-}6 = 48$	$^{-}9 \cdot 9 = -81$
$9 \cdot 5 = 45$	$8 \cdot {}^{-}8 = -64$	$^{-}9 \cdot {}^{-}6 = 54$
$^{-}9 \cdot 5 = -45$	$^{-}3 \cdot 5 = -15$	$4 \cdot {}^{-}8 = -32$

$4 \cdot 6 = 24$ $6 \cdot 4 = 24$	$9 \cdot {}^{-}7 = -63$ $^{-}7 \cdot 9 = -63$	$^{-}1 \cdot 16 = -16$ $16 \cdot {}^{-}1 = -16$	$^{-}5 \cdot {}^{-}1 = 5$ $^{-}1 \cdot {}^{-}5 = 5$
$^{-}5 \cdot 8 = -40$ $8 \cdot {}^{-}5 = -40$	$^{-}3 \cdot {}^{-}7 = 21$ $^{-}7 \cdot {}^{-}3 = 21$	$^{-}12 \cdot 0 = 0$ $0 \cdot {}^{-}12 = 0$	$^{-}8 \cdot {}^{-}9 = 72$ $^{-}9 \cdot {}^{-}8 = 72$

It doesn't matter which of the numbers comes first when we are multiplying, so we say that multiplication of integers is commutative.

Commutative Principle for Multiplication of Integers:
If a and b are integers, then $a \cdot b = b \cdot a$.

For example, $\underbrace{{}^{-}3 \cdot 4}_{{}^{-}12} = \underbrace{4 \cdot {}^{-}3}_{{}^{-}12}$.

Multiply.

$^-3(9) = -27$	$(6)(^-5) = -30$
$^-8(^-9) = 72$	$(^-4)(^-6) = 24$
$7(8) = 56$	$(^-5)(7) = -35$
$^-6(^-6) = 36$	$(9)(9) = 81$
$1(9) = 9$	$9(7) = 63$
$^-10 \cdot {}^-9 = 90$	$9 \cdot 7 = 63$
$(^-8)(1) = -8$	$(9)(7) = 63$
$6(^-8) = -48$	$\underbrace{(3 \cdot 3) \cdot 7}_{9 \cdot 7} = 63$
$(2)(43) =$	$\underbrace{3 \cdot (3 \cdot 7)}_{3 \cdot 21} = 63$

Here are three different ways to write a multiplication problem:

$$3 \cdot {}^-7 = {}^-21$$

$$3(^-7) = {}^-21$$

$$(3)(^-7) = {}^-21$$

All of them say:
"3 *times* ⁻7 *equals* ⁻21."

$\overbrace{(^-3 \cdot 5)}^{-15} \cdot 4 = -60$ $^-3 \cdot \underbrace{(5 \cdot 4)}_{20} = -60$	$\overbrace{(^-6 \cdot 8)}^{-48} \cdot {}^-2 = 96$ $^-6 \cdot \underbrace{(8 \cdot {}^-2)}_{-16} = 96$	$\overbrace{(^-3 \cdot {}^-2)}^{6} \cdot {}^-5 = -30$ $^-3 \cdot \underbrace{(^-2 \cdot {}^-5)}_{10} = -30$
$\overset{-72}{(5 \cdot {}^-8)} \cdot {}^-2 = 144$ $5 \cdot \underset{16}{(^-8 \cdot {}^-2)} = 144$	$\overset{4}{(^-2 \cdot {}^-2)} \cdot {}^-2 = -8$ $^-2 \cdot \underset{4}{(^-2 \cdot {}^-2)} = -8$	$\overset{25}{(^-5 \cdot {}^-5)} \cdot 2 = 50$ $^-5 \cdot \underset{-10}{(^-5 \cdot 2)} = 50$

It doesn't matter which pair of numbers we multiply first. The answer comes out the same either way, so we say that multiplication of integers is associative.

Associative Principle for Multiplication of Integers:
If a, b and c are integers, then $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

For example, $\underbrace{(5 \cdot {}^-2) \cdot 3}_{^-30} = \underbrace{5 \cdot (^-2 \cdot 3)}_{^-30}$

$\underbrace{2 \cdot 6}_{12} \cdot {}^-3 = {}^-36$

$\underbrace{(3)(7)(^-2)}_{21\ (-2)} = -42$

$\underbrace{{}^-5 \cdot 6 \cdot {}^-3}_{-30\, \cdot\, -3} = 90$

$\underbrace{4 \cdot {}^-5 \cdot 3}_{-20\, \cdot\, 3} = -60$

$\underbrace{(^-3)(^-3)(^-3)}_{9\ (-3)} = -27$

$\underbrace{8 \cdot {}^-2 \cdot {}^-4}_{-16\, \cdot\, -4} = 64$

$\underbrace{{}^-8 \cdot {}^-2 \cdot {}^-2}_{16\, \cdot\, -2} = -32$

$\underbrace{(5)(^-10)(2)}_{-50(2)} = -100$

$\underbrace{{}^-1 \cdot {}^-1 \cdot 12}_{1\, \cdot\, 12} = 12$

$\underbrace{6 \cdot {}^-5 \cdot {}^-3}_{-30\, \cdot\, -3} = 90$

$\underbrace{(4)(4)(4)}_{16} = 64$

$\underbrace{46 \cdot {}^-37 \cdot 0}_{-} = 0$

Multiply.

$6 \cdot 0 = 0$	$0 \cdot 7 = 0$	$8 \cdot 1 = 8$	$1 \cdot 3 = 3$	$5 \cdot {}^-1 = -5$	${}^-1 \cdot 7 = -7$
$8 \cdot 0 = 0$	$0 \cdot {}^-3 = 0$	${}^-6 \cdot 1 = -6$	$1 \cdot {}^-5 = -5$	$9 \cdot {}^-1 = -9$	${}^-1(18) = -18$
${}^-4 \cdot 0 = 0$	$0 \cdot 0 = 0$	$15 \cdot 1 = 15$	$1 \cdot 1 = 1$	${}^-4 \cdot {}^-1 = 4$	${}^-1 \cdot {}^-6 = 6$
${}^-13 \cdot 0 = 0$	$0(16) = 0$	${}^-16 \cdot 1 = -16$	$1 \cdot {}^-17 = -17$	$53 \cdot {}^-1 = -53$	${}^-1 \cdot 0 = 0$
$48 \cdot 0 = 0$	$0 \cdot 1 = 0$	${}^-63 \cdot 1 = -63$	$1(26) = 26$	${}^-1 \cdot {}^-1 = 1$	${}^-1(^-23) = 23$
$(^-29)(0) =$	$0 \cdot {}^-58 = 0$	$80 \cdot 1 = 81$	$1 \cdot 0 = 0$	${}^-8 \cdot {}^-1 = 8$	${}^-1 \cdot 1 = -1$

Sometimes multiplying can be pretty hard to do — but not when we are multiplying by 0, 1 or ⁻1. Then it's very simple. Here are the principles that tell us what to do:

Principle for Multiplying by Zero:
If a is any integer, then $a \cdot 0 = 0$
and $0 \cdot a = 0$.

Principle for Multiplying by One:
If a is any integer, then $a \cdot 1 = a$
and $1 \cdot a = a$.

Principle for Multiplying by Negative One:
If a is any integer, then $a \cdot {}^-1 = -a$
and ${}^-1 \cdot a = -a$.
"Any integer times negative one is the opposite of the integer."

Order of Operations

We saw that parentheses are often used in algebra to show what to do first. Look at this problem:

$$4 \cdot (5 + 2) =$$

The parentheses tell us to add $5 + 2$ first, and then to multiply the answer by 4:

$$4 \cdot \underbrace{(5 + 2)}_{7} = 28$$

Below are some problems for you to do.

$5 \cdot \underbrace{(2+3)}_{5 \cdot (5)} = 25$

$\underbrace{(5 \cdot 2)}_{(10)+3} + 3 = 13$

$7 - \underbrace{(5-2)}_{7-(3)} = 4$

$* \underbrace{(7-5)}_{(2)-2} - 2 = -4$

$\underbrace{(^-6+4)}_{(-2)\cdot 3} \cdot 3 = -6$

$^-6 + \underbrace{(4 \cdot 3)}_{-6+(12)} = 6$

$5 + \underbrace{(2 \cdot 3)}_{5+6} = 11$

$\underbrace{(5+2)}_{7 \cdot 3} \cdot 3 = 21$

$7 + \underbrace{(5+2)}_{7+(7)} = 14$

$\underbrace{(7+5)}_{(12)+2} + 2 = 14$

$\underbrace{(^-2 \cdot {}^-2)}_{(4)+6} + 6 = 10$

$^-2 \cdot \underbrace{(^-2+6)}_{-2 \cdot (4)} = -8$

$\underbrace{(4-3)}_{(1)} \cdot \underbrace{(5-1)}_{(4)} = 4$

$\underbrace{(3-5)}_{(-2)} \cdot \underbrace{(10-6)}_{(4)} = -8$

$\underbrace{(5-3)}_{(2)} \cdot \underbrace{(6-10)}_{(-4)} = -8$

$\underbrace{(3-5)}_{(-2)} \cdot (6-10) = 8$ $(-2) \cdot (-4)$

$\underbrace{(5-3)}_{(2)} \cdot \underbrace{(10-6)}_{(4)} = 8$

$\underbrace{(3 \cdot 4)}_{(12)} + \underbrace{(4 \cdot 2)}_{(8)} + \underbrace{(3 \cdot 3)}_{(9)} = 29$

$\underbrace{(^-3 \cdot 4)}_{(1)} + \underbrace{(4 \cdot 2)}_{(8)} + \underbrace{(^-3 \cdot {}^-3)}_{(9)} = 18$

$\underbrace{(^-3 \cdot {}^-4)}_{(12)} + \underbrace{(^-4 \cdot 2)}_{(-8)} + \underbrace{(3 \cdot {}^-3)}_{(-9)} = -5$

$\underbrace{(^-3 \cdot {}^-4)}_{(12)} + \underbrace{(^-4 \cdot {}^-2)}_{(8)} + \underbrace{(^-3 \cdot {}^-3)}_{(9)} = 29$

$\underbrace{(^-3 \cdot 4)}_{(-12)} + \underbrace{(4 \cdot {}^-2)}_{(-8)} + \underbrace{(^-3 \cdot 3)}_{(-9)} = -29$

Here is a problem that doesn't have any parentheses to show what to do first:

$$5 + 3 \cdot 4$$

There are two ways you could try to do this problem.

Multiplying first: $5 + \underbrace{3 \cdot 4}_{12} = 17$ C

Adding first: $\underbrace{5 + 3}_{8} \cdot 4 = 32$ X

As you can see, the answers are different. The first one, 17, is right because of a rule we always follow in doing computations:

1. If there are *parentheses*, first do what is in them.
2. Then do all the *multiplying*, from left to right.
3. Finally, do the rest of the *adding and subtracting*, from left to right.

See if you can follow this rule on each problem below.

$\underbrace{(8 + 5)}\cdot 2 = 26$
$(13) \cdot 2$

$8 + \underbrace{5 \cdot 2} = 18$
$8 + 10$

$\underbrace{7 \cdot 3} + 10 = 31$
$21 + 10$

$\underbrace{(6 + 2)} \cdot 4 = 32$
$8 \cdot 4$

$\underbrace{5 \cdot 6} - 12 = 18$
$30 - 12$

* $25 - \underbrace{3 \cdot 6} = 7$
$25 - 18$

$3 \cdot \underbrace{(6 + 4)} = 30$
$3 \cdot 10$

$\underbrace{(5 + {}^{-}7)} \cdot 2 = -4$
$(-2) \cdot 2$

$5 + \underbrace{{}^{-}7 \cdot 2} = 0$
$5 + (-5)$

$8 \cdot \underbrace{(2 - 4)} = -16$
$8 \cdot (-2)$

$\underbrace{8 \cdot 2} - 4 = 12$
$16 - 4$

${}^{-}5 + \underbrace{3 \cdot {}^{-}2} = -11$
$-5 + (-6)$

$\underbrace{{}^{-}5 \cdot 3} + {}^{-}2 = -17$
$-15 + -2$

${}^{-}2 + \underbrace{{}^{-}2 \cdot {}^{-}2} = 2$
$-2 + 4$

$7 + \underbrace{(6 - 2)} \cdot 5 = 27$
$7 + \underbrace{(4) \cdot 5}$
$7 + 20$

$16 + \underbrace{3 \cdot 7} - 12 = 25$
$\underbrace{16 + 21} - 12$
$37 - 12$

* $\underbrace{6 \cdot 4} + \underbrace{5 \cdot 4} = 44$
$24 + 20$

$8 + 4 \cdot \underbrace{(5 + 2)} = 32$
$8 + \underbrace{4 \cdot 7}$
$8 + 24$

$5 \cdot \underbrace{(7 - 3)} - 6 = 14$
$\underbrace{5 \cdot 4} - 6$
$20 - 6$

$9 - \underbrace{(3 + 4)} - 2 = 0$
$\underbrace{9 - 7} - 2$
$2 - 2$

$12 + 5 \cdot \underbrace{(8 - 2)} = 42$
$12 + \underbrace{5 \cdot 6}$
$12 + 30$

$\overbrace{3\cdot 5}^{15} + \overbrace{4\cdot 5}^{20} = \mathbf{35}$ $\underbrace{(3+4)}_{7}\cdot 5 = \mathbf{35}$	$\overbrace{6\cdot 2}^{12} + \overbrace{5\cdot 2}^{10} = 22$ $\underbrace{(6+5)\cdot 2}_{11\cdot 2} = 22$	$\overbrace{3\cdot 5}^{15} + \overbrace{3\cdot 2}^{6} = 21$ $\underbrace{3\cdot(5+2)}_{3\cdot 7} = 21$
$\overbrace{5\cdot 10}^{50} + \overbrace{4\cdot 10}^{40} = 90$ $\underbrace{(5+4)\cdot 10}_{9\cdot 10} = 90$	$\overbrace{3\cdot {}^{-}4}^{-12} + \overbrace{4\cdot {}^{-}4}^{-16} = -28$ $\underbrace{(3+4)\cdot {}^{-}4}_{7\cdot -4} = -28$	$\overbrace{7\cdot 3}^{21} + \overbrace{7\cdot 7}^{49} = 70$ $\underbrace{7\cdot(3+7)}_{7\cdot 10} = 70$
$\overbrace{({}^{-}4)(3)}^{-12} + \overbrace{({}^{-}5)(3)}^{-15} = -27$ $\underbrace{({}^{-}4+{}^{-}5)(3)}_{(-9)(3)} = -27$	$\overbrace{({}^{-}7)({}^{-}5)}^{35} + \overbrace{(4)({}^{-}5)}^{-20} = 15$ $\underbrace{({}^{-}7+4)({}^{-}5)}_{(-3)(-5)} = 15$	$\overbrace{{}^{-}2\cdot 4}^{-8} + \overbrace{{}^{-}2\cdot 6}^{-12} = -20$ $\underbrace{{}^{-}2\cdot(4+6)}_{-2\cdot 10} = -20$

Were you surprised to see the answers come out the same in each pair of problems? If you were, then try thinking about it like this:

$$\overset{\text{3 fives}}{\boxed{3\cdot 5}} + \overset{\text{4 fives}}{\boxed{4\cdot 5}}$$

$$\underset{\text{7 fives}}{\boxed{(3+4)\cdot 5}}$$

Doesn't it make sense for the answers to come out the same? After all, 3 fives and 4 fives is equal to 7 fives. Since this works for any integers we may choose, we say that multiplication of integers is **distributive** over addition.

> Distributive Principle:
> If a, b and c are integers, then $(b+c)\cdot a = b\cdot a + c\cdot a$
> and $a\cdot(b+c) = a\cdot b + a\cdot c$.

You've learned two ways to solve this problem:

$$5(3+8) = $$

On page 30 you learned to do what is in the parentheses first, then multiply:

$$5(\overbrace{3+8}) = 5(11) = 55$$

And on page 31 you learned that using the Distributive Principle gives the same answer:

$$5(3+8) = 5(3) + 5(8) = 15 + 40 = 55$$

When we use the Distributive Principle we can save some work by doing the multiplications in our head and just writing the products:

$$5(3+8) = 15 + 40 = 55$$

Do each problem two ways.

$6(\overbrace{4+6}) = \mathbf{6(10) = 60}$ $6(4+6) = \mathbf{24 + 36 = 60}$	$7(\overbrace{20+8}^{7(28)}) = 196$ $7(\underbrace{20+8}_{140+56}) = 196$
$3(\overbrace{8+2}^{3(10)}) = 30$ $3(\underbrace{8+2}_{24+6}) = 30$	$8(\overbrace{^-4+1}^{8(-3)}) = -24$ $8(\underbrace{^-4+1}_{-32+8}) = -24$
$^-4(\overbrace{9+^-3}^{-4(6)}) = -24$ $^-4(\underbrace{9+^-3}_{-36+12}) = -24$	$^-3(\overbrace{30+^-2}^{-3(28)}) = -84$ $^-3(\underbrace{30+^-2}_{-90+6}) = -84$

$$3(2-6) = \mathbf{3(2+^-6) = 3(^-4) = ^-12}$$

$$3(2-6) = \mathbf{3(2+^-6) = 6+^-18 = ^-12}$$

$$10(3-8) = 10(3+-8) = 10(-5) = -50$$

$$10(3-8) = 10(3+-8) = 30+-80 = -50$$

$$^-2(^-4-7) = -2(-4+-7) = -2(-11) = 22$$

$$^-2(^-4-7) = -2(-4+-7) = 8+14 = 22$$

Dividing Integers

In division problems, the number we divide *by* is called the **divisor** and the number we divide *into* is called the **dividend**.

$$12 \div 4 = 3 \qquad\qquad 4\overline{)12}^{\,3}$$

divisor

dividend

When you divide integers you have to break down the problem into two parts. We find the *amount* of the answer by dividing, and we find the *sign* of the answer by following these rules:

POSITIVE	•	POSITIVE	=	POSITIVE
POSITIVE	•	NEGATIVE	=	NEGATIVE
NEGATIVE	•	POSITIVE	=	NEGATIVE
NEGATIVE	•	NEGATIVE	=	POSITIVE

These rules are the same as the rules for multiplication. That's because the answer to a division problem can be found by reversing a multiplication problem.

$18 \div 3 = 6$	because	6	times 3	is	18.
$18 \div {}^{-}3 = {}^{-}6$	because	${}^{-}6$	times ${}^{-}3$	is	18.
${}^{-}18 \div 3 = {}^{-}6$	because	${}^{-}6$	times 3	is	${}^{-}18$.
${}^{-}18 \div {}^{-}3 = 6$	because	6	times ${}^{-}3$	is	${}^{-}18$.

Here are some division problems for you:

$12 \div 3 = 4$	$10 \div 5 = 2$	${}^{-}18 \div 6 = -3$
${}^{-}12 \div {}^{-}3 = 4$	${}^{-}10 \div {}^{-}5 = 2$	$18 \div 6 = 3$
$12 \div {}^{-}3 = {}^{-}4$	$10 \div {}^{-}5 = -2$	$18 \div {}^{-}6 = -3$
${}^{-}12 \div 3 = {}^{-}4$	${}^{-}10 \div 5 = -2$	${}^{-}18 \div {}^{-}6 = 3$
${}^{-}7 \div {}^{-}7 = 1$	${}^{-}15 \div 1 = -15$	$-9 \div -9 = 1$
${}^{-}7 \div 7 = -1$	${}^{-}15 \div {}^{-}1 = 15$	$-9 \div 9 = -1$
$7 \div 7 = 1$	$15 \div {}^{-}1 = -15$	$0 \div 9 = 0$
$7 \div {}^{-}7 = -1$	$15 \div 1 = 15$	$9 \div 0 = 0$

Did you get the last two problems? If not, the next page may help you.

$12 \div 4 =$ What number times 4 equals 12? <u>3</u>

$12 \div 4 = 3$

$0 \div 9 =$ What number times 9 equals 0? <u>0</u>

$0 \div 9 = 0$

$9 \div 0 =$ What number times 0 equals 9? <u>none</u>

This problem has no answer.

In fact, no number divided by zero has an answer. That's why we say:

> We can *never* divide by 0.

Do these problems. If a problem has no answer, cross it out.

$(7-7) \div (4+6) = 0 \div 10 = 0$

$\cancel{(4+6) \div (7-7)} = \cancel{10 \div 0}$

$\cancel{(^-8+2) \div (^-3+3)} = \cancel{-6 \div 0}$

$(^-3+3) \div (^-8+2) = 0 \div (-6) = 0$

$(12-4) \div (4-12) = 8 \div (-8) = -1$

$(4-12) \div (12-4) = (-8) \div 8 = -1$

$\cancel{(5+5) \div (5-5)} = \cancel{10 \div 0}$

$(^-3+^-2) \div (^-3+2) = (-5) \div (-1) = 5$

$(^-6+^+6) \div (5+2) = 0 \div 7 = 0$

$\cancel{(7+7) \div (^-7+^+7)} = \cancel{14 \div 0}$

$(6+4) \div (4+^-6) = 10 \div (-2) = -5$

$(11+^-11) \div (^-3+^-3) = 0 \div -6 = 0$

IMPORTANT NOTICE

There are two big differences between dividing integers and the other operations you have learned (adding, subtracting and multiplying).

1. We can *never* divide by zero.
2. If the divisor doesn't go into the dividend evenly, then the answer will not be an integer. For example, $10 \div 3$ does not equal any integer. We will discuss problems like this when we study rational numbers in Book 5.