Review of Operations on Integers

In Book 1 you were told about numbers called integers — positive integers, negative integers and zero. You learned how to add, subtract, multiply and divide integers.

Adding Integers: Think of positive integers as gains, negative integers as losses and zero as showing no change. To add two or more integers, just think of the integer which shows the overall change.

Subtracting Integers: Think of the problem as an adding problem — only be sure to add the opposite of the number you are subtracting.

Multiplying and Dividing Integers: Break the problem down into two parts. Find the amount by multiplying or dividing. The sign will be positive if you are multiplying or dividing two numbers with the same sign. The sign will be negative if the two numbers you are multiplying or dividing have different signs.

Below are some problems for you to do. Some are adding, some are subtracting, some are multiplying and some are dividing — so be careful . . .

$$-5 + -3 = -8$$
 $-6 \cdot -9 = 54$ $-8 + -8 = -16$ $-8 \cdot 1 = -8$
 $-8 \cdot 5 = -40$ $-5 + +5 = 0$ $-10 \cdot 0 = 0$
 $5 \cdot -7 = -35$ $-24 \div 3 = -8$ $-18 \div -9 = 2$ $-3 \div 3 = -6$
 $-6 \div 4 = -10$ $-5 \cdot -5 = 25$ $-6 \div 4 = 0$ $-6 \div 6 = 0$
 $-6 \div 2 = -10$ $-6 \div 9 = 54$ $-6 \cdot 10 = 0$ $-6 \div 10 = 0$ $-6 \div 10 = 0$

$$10 + 76 = -6$$
 $14 + 74 = 0$ $1 \cdot 7 = 7$ $-(-3) = 3$

$$(10)(-16) = -160 (-3)(3) = -9 0 - (5) = -5$$

$$5+14=-9$$
 $14+6=8$ $-5+-5=-10$ $-(-9)=9$

$$-1.6 = -6$$
 $|\cdot| = |$ $(-8)(-1) = 8$ $-(0) = 0$

These problems take more than one step:

$$5 \cdot 3 - 2 = 13$$
 $4 + (6 - 7) = 3$ $-8 + 3(-2) = -14$
 $5 \cdot (3 - 2) = 5$ $4 + 6 - 7 = 3$ $(-8 + 3)(-2) = 10$
 $5 \cdot (1)$ $(-5)(-2)$

Here are some problems that follow a pattern:

To write the pattern for a group of problems just copy down the part that is always the same and use a letter in place of each number that changes. Here is the pattern for the problems above:

Letters that are used where numbers can go are called **variables**. Patterns are also called **expressions**. Here are some examples:

Variables: x a t b y Expressions: $2 \cdot x$ x + 4 $3 \cdot x + 4$ x - 5 x + y $3 \cdot (a + 6)$

We usually don't write the multiplication dots in expressions if the meaning is clear without them. For example,

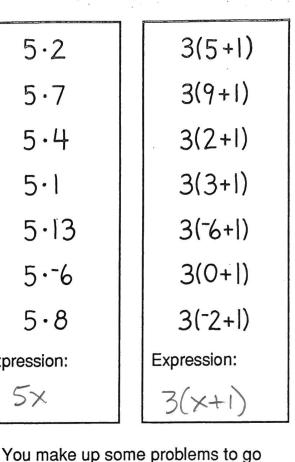
 $x \cdot y$ is written xy $2 \cdot x$ is written 2x $3 \cdot x + 4$ is written 3x + 4 $3 \cdot (a+6)$ is written 3(a+6) $4 \cdot (x-3)$ is written $\frac{2n+5}{4(x-3)}$

Write an expression for each group of problems.

The second second second	3.5 + 2
	3.6 + 2
	3.8+2
	3-4+2
	3 3 + 2
	3 2 + 2
	3-10+2

Expression: 3a+2

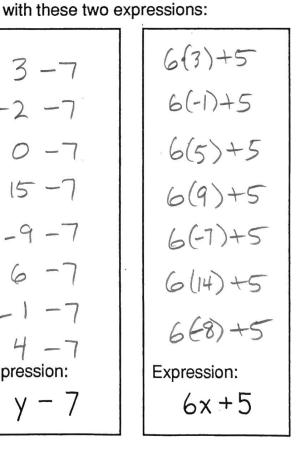
reach group of pr				
8+4				
3 + 4				
0+4				
-3 + 4				
4 + 4				
13/+ 4				
10 + 4				
Expression:				
X+4				



5-2.8 5-2.7 5-2.4 5-2.0 5-2-3 5-2.10 5-2-8 Expression:

$$-(-4) + 3$$

 $-(6) + 3$
 $-(-8) + 3$
 $-(-5) + 3$
 $-(2) + 3$
 $-(15) + 3$
 $-(-6) + 3$
Expression:
 $-x + 3$



5-2×

$$x + 10$$

In order to make up problems that follow this pattern we just substitute different numbers for the variable.

Substitute 3 for
$$x$$
:

$$7 + 10 = 17$$

Substitute
$$-5$$
 for x : $-5 + 10 = 5$

You substitute the given numbers in each expression below.

Substitute 7 for y:
$$7-4=3$$

$$7 - 4 = 3$$

Substitute 5 for y:
$$5-4=1$$

Substitute 4 for y:
$$4-4=0$$

Substitute 3 for y:
$$3-4=-1$$

Expression: 3x

Substitute 5 for
$$x$$
: 3(5) = 15

Substitute 6 for
$$x$$
: $3(6) = 1\%$

Substitute 7 for
$$x$$
:

Substitute 7 for
$$x: 3(7) = 2$$

Substitute 8 for x:
$$3(8)=24$$

Expression:
$$5x + 2$$

$$5x + 2$$

Substitute 4 for x:
$$5.4+2 =$$

$$5.4 + 2 =$$

Substitute 5 for x:
$$5(5)+2=27$$

Substitute
$$-5$$
 for x :

Substitute
$$-5$$
 for x : $5(-5)+2=-23$

Substitute 0 for
$$x$$
:

Substitute 0 for
$$x$$
: $5(0)+2=2$

and 7 for γ :

$$x + y$$

and 4 for y:

$$6+(-4)=2$$

Substitute 9 for
$$x$$

,	Expression:	2× + 3	Expression:	2x + 3
,	Substitute 0 for x :	2(0)+3=3	Substitute ⁻¹ for <i>x</i> :	2(-1)+3=
_	Substitute 1 for x:	2(1)+3=5	Substitute ⁻ 2 for <i>x</i> :	2(-2)+3=-
	Substitute 2 for x:	2(2)+3=7	Substitute $\bar{\ }$ 3 for x :	2(-3)+3=-3
	Substitute 3 for x:	2(3)+3=9	Substitute $^{-4}$ for x :	2(-4)+3=-5
′	Substitute 4 for x:	2(4)+3=11	Substitute $^{-}5$ for x :	2(-5)+3=-7

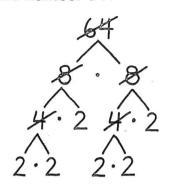
Often it's easier to show something in the form of a **table**. Here are some **substitution tables** for you to finish:

χ	2 x	· ×	$\chi + \chi$	X	-×
3	2(3)=6	3	3+3=6	⁻ 5	-(⁻ 5)=5
5	2(5) = 10	5	5+5= 10	5	-(5)= ⁻ 5
8	2(8)=16	8	8+8=16	-8	-(-8) = 8
10	2(10) = 20	10	10+10=Z0	_8_	-(8)=-8
-5	2(-5) = -10	⁻ 5	-5+-5=-10	_6	-(6) = -6
	200			14	-(14) = -14
*	ž.			0	-(0)=0

Each table below has two variables.

χ	У	x + y	χ	У	ху
3	4	3+4=7	3	4	(3)(4)=12
5		5+2 = 7	5	2	(5)(2)=10
2	3	2+3=5	2	3	(2)(3) = 6
-6	3	-6+3=-3	-6	3	(-6)(3) = -18

Remember when we factored the number 64?



$$64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

Here is an easier way to write the answer using an exponent:

$$64 = 2^6$$
 exponent base

Do you see what the exponent 6 stands for? It tells how many 2's we have to multiply together to get 64.

Here are some more examples of how we can use exponents:

$$5 \cdot 5 = 5^2$$
 "5 squared"
 $5 \cdot 5 \cdot 5 = 5^3$ "5 cubed"
 $5 \cdot 5 \cdot 5 \cdot 5 = 5^4$ "5 to the 4th power"
 $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^5$ "5 to the 5th power"

Now finish filling in this table:

6.6	6°	"6 squared"
3.3.3.3.3	3⁵	"3 to the 5th power"
3.3.3.3	34	3 to the 4th power
7.7	72	7 Squared
2 · 2 · 2	23	2 cubed
8-8-8-8-8	86	8 to the 6th power

	Exponential Form	Factored Form	Multiplied Form
4 oguarad	LL ²	L · L	16
4 squared	•		
5 squared	52	5.5	25
3 squared	. 62	6.6	36
⁻ 4 squared	(-4) ²	-44	16
⁻ 5 squared	$(-5)^2$	-55	25
⁻ 6 squared	$(-6)^2$	-66	36
2 cubed	2³	2 · 2 · 2	8
3 cubed	3 ³	3.3.3	27
4 cubed	43	4.4.4	64
² cubed	(⁻ 2) ³	-222	-8
3 cubed	(-3)3	-333	-27
4 cubed	(4)3	-444	-64
2 to the 4th power	24	2.2.2.2	16
3 to the 4th power	34	3.3.3.3	81
2 to the 4th power	(-2)4	-222	16
3 to the 4th power	$(-3)^{4}$	-333	81
2 to the 5th power	Z ⁵	2.2.2.2.2	32
3 to the 5th power	35	3.3.3.3.3	243
2 to the 5th power	$(-2)^5$	-2222	-32
3 to the 5th power	(-3)5	-3333	-243
2 to the 6th power	26	2.2.2.2.2.2	64 .
3 to the 6th power	3	3.3.3.3.3	729
2 to the 7th power	27	2.2.2.2.2.2	128

$$xxxxx = x^5$$

$$aaa = a^3$$

$$xxx = X^3$$

$$(ab)(ab)(ab) = (ab)^3$$

$$(xy)(xy)(xy)(xy) = (xy)^4$$

$$(3x)(3x)(3x) = (3x)^3$$

$$(x+2)(x+2) = (x+2)^2$$

$$(x+2)(x+2)(x+2) = (x+2)^3$$

$$mm = w^2$$

$$(-8a)(-8a)(-8a)(-8a) = (-8a)^4$$

$$(4xyz)(4xyz)(4xyz) = (4xyz)^3$$

$$(5a)(5a) = (5a)^2$$

$$(x+4)(x+4) = (x+4)^{2}$$

$$(a-b)(a-b)(a-b)(a-b) = (a-b)^4$$

Write out each expression the long way.

$$x^4 = xxxx$$

$$b^2 = bb$$

$$(4x)^3 = (4x)(4x)(4x)$$

$$(4x)^4 = (4x)(4x)(4x)(4x)$$

$$(xy)^2 = (xy)(xy)$$

$$y^3 = y y y$$

$$\chi^2 = \times \times$$

$$(x+5)^2 = (x+5)(x+5)$$

$$(x-3)^4 = (x-3)(x-3)(x-3)(x-3)$$

Use exponents to shorten each expression.

Write out each expression the long way.

$$6x^{5}y^{2} = 6xxxxxyy$$
 $(ab)^{3} = (ab)(ab)(ab)$
 $8a^{3}b^{2} = 8aaabb$ $(xy)^{4} = (xy)(xy)(xy)(xy)$
 $x^{2}y^{5} = xxyyyyy$ $x^{4}y^{4} = xxxxyyyy$
 $12x^{4}y = 12xxxxy$ $(2x)^{3}(5y)^{2} = (2x)(2x)(2x)(5y)(5y)$
 $x^{3}y^{2}z^{3} = xxxyyzzz$ $2x^{4} = 2xxxx$
 $a^{3}b^{2}c = aaabbc$ $(2x)^{4} = (2x)(2x)(2x)(2x)$

Here are the answers to the last two problems:

$$2x^{4} = 2xxxx$$

 $(2x)^{4} = (2x)(2x)(2x)(2x)$

Did you get them right? Do you see why the answers have to be different? If you pay attention to parentheses you shouldn't have any trouble writing these out the long way:

$$5a^{4} = 5aaaa$$
 $(7x)^{2} = (7x)(7x)$
 $(5a)^{4} = (5a)(5a)(5a)(5a)$ $7x^{2} = 7xx$
 $6ab^{3} = 6abbb$ $(xyz)^{2} = (xyz)(xyz)$
 $6(ab)^{3} = 6(ab)(ab)(ab)$ $x(yz)^{2} = xyz$
 $(6ab)^{3} = (6ab)(6ab)(6ab)$ $xyz^{2} = xyzz$

×	χ ²
	2
2	22=2:2=4
3	32=3.3=9
4	42=4.4-16
5	5 ² -5.5=25
6	62-6-6-36
7	72=7.7=49
8	82=8.8=64
9	$9^2 = 9.9 = 81$
10	102=10.10=100
11	$ 1^2 = 1.11 = 2 $
ı	

$$\begin{array}{c|ccccc} x & x^{3} \\ \hline -1 & (-1)^{3} = (-1)(-1)(-1) = -1 \\ \hline -2 & (-2)^{3} = (-2)(-2)(-2) = -8 \\ \hline -3 & (-3)^{3} = (-3)(-3)(-3) = -2.7 \\ \hline -4 & (-4)^{3} = (-4)(-4)(-4) = -64 \\ \hline -5 & (-5)^{3} = (-5)(-5)(-5) = -125 \\ \hline -6 & (-6)^{3} = (-6)(-6)(-6) = -216 \\ \hline \end{array}$$

quivalent Expressions

lere are two expressions:

$$6x^2$$

Let's see what happens when we substitute several numbers for x.

X	(3x)(2x)	χ	6x²
4	(3.4)(2.4)=12.8=96	4	$6(4)^2 = 6 \cdot 16 = 96$
10	$(3 \cdot 10)(2 \cdot 10) = 30 \cdot 20 = 600$	10	$6(10)^2 = 6 \cdot 100 = 600$
-2	(3-2)(2-2) = -6-4 = 24	-2	$6(-2)^2 = 6.4 = 24$
5	(3.5) (2.5) = 15.10=150	5	$6(5)^2 = 6 \cdot 25 = 150$
3	(3.3)(2.3) = 9.6 = 54	3	$6(3)^2 = 6.9 = 54$

fou can try substituting some more numbers, but you will find that no matter what number ou try, the answers in the two tables always come out the same. We say that (3x)(2x) and $6x^2$ are **equivalent expressions**. This shouldn't surprise you since you already now that multiplication of integers is associative and commutative, so

$$(3x)(2x) = (3.2)(xx) = 6x^2$$

Any times in algebra you will be asked to **simplify** an expression. All this means is that ou are supposed to find an equivalent expression that's easier to write.

implify each expression below.

$$(4x)(5x) = (4.5)(xx) = 20x^{2}$$

 $(3a)(6a) = (3.6)(aa) = 18a^{2}$
 $(5y)(5y) = (5.5)(yy) = 25y^{2}$
 $(2x)(7x) = (2.7)(xx) = 14x^{2}$

A term is a very simple kind of expression where multiplication is the only operation. Here are some examples of terms:

$$5x 3a^2 8xy x 24a^4bc^3 7$$

Most terms have two parts — a number part and a variable part. For example, 5 is the number part of 5x, and x is the variable part of 5x. The number part is sometimes called the **coefficient**.

When we are multiplying terms, it is easiest to break the problem down into steps. First multiply the number parts of all the terms together. Then multiply the variable parts together.

$$(4x)(-5x) = (4-5)(xx) = -20x^{2}$$

$$(4x)(-5x) = -20x^{2}$$

$$(7a)(2a) = |4a^{2}|$$

$$(-3x)(5x) = -15x^2$$

$$(4y)(-3y) = -12y^2$$

$$(2x)(4y) = (2 \cdot 4)(xy) = 8xy$$

$$(2x)(4y) = 8xy$$

$$(5a)(2b) = |0ab$$

$$(-4x)(-2y) = 8xy$$

$$(10a)(4b) = -40ab$$

$$(7u)(-3v) = -2|uv$$

(-7u)(-3v) = 2 |uv|

$$(6c)(4c) = 24c^{2}$$

$$(10a)(-2a) = -20a^{2}$$

$$(6x)(5x) = 30x^{2}$$

$$(-7w)(3w)(2w) = -7 \cdot 3 \cdot 2w^{3} = -42w^{3}$$

$$(2a)(-5a)(-5a) = 2 \cdot -5 \cdot -5a^{3} = 50a^{3}$$

$$(x)(7x)(2x) = 14x^{3}$$

$$(-1v)(4v)(2v) = -8v^{3}$$

$$(6u)(-8v) = -48uv$$

 $(6u)(8v) = 48uv$
 $(5x)(2y)(3z) = 30xyz$
 $(-6a)(b)(7c) = -42abc$
 $(-3u)(-3v)(-3w) = -27uvw$
 $(2a)(5b)(2c)(3d) = 60abcd$
 $(5t)(-10u)(-3v) = 150tuv$

implify.

When you multiply with veriables, you add the exponents.

$$(5x^2)(3x^3) = (5xx)(3xxx) = (5\cdot3)(xxxxx) = 15x^5$$

$$(5x^2)(3x^3) = 15x^5$$

$$(6x^3)(2x^4) = 12x^7$$

$$(5a^2)(-5a^4) = -25a^6$$

$$(-9x)(-4x^3) = 36x^4$$

$$(3x)(5x^3)(4x^2) = 60x^6$$

$$(-4n)(-2n^2)(n) = 8n^4$$

$$(6a^4)(6a^4) = 36a^8$$

$$(-2a^2)(-2a^2)(-2a^2) = -8a^6$$

 $(-9x^{2}y)(8x^{2}y^{4}) = (-9xxy)(8xxyyyy) = -72x^{4}y^{5}$

$$(-9x^2y)(8x^2y^4) = -72x^4y^5$$

$$(4a^2b^3)(7a^2b) = 28a^4b^4$$

$$(-3x^2y^3)(-9x^3y^5) = 27x^5y^8$$

$$(9x^2)(6y^2) = 54x^2y^2$$

$$(3x^4y)(7x^3y) = 21 \times^7 y^2$$

$$(-5xy)(-9xy)(-2xy) = -90x^3y^3$$

$$(5mn)(-9m^3n) = -45m^4n^2$$

$$(-4x)(-4x)(-4x) = -64x^3$$

$$(3xy)(3xy)(3xy) = 27x^3y^3$$

$$(2x^3)(2x^3)(2x^3)(2x^3) = (6x^{12})$$

$$(3x^2)(3x^2)(3x^2)(3x^2) = 8x^8$$

$$(2x)(2x)(2x)(2x)(2x)(2x)(2x)(2x)(2x) = 2^9 x^9 = 512 x^9$$

$$(2x)(5y)(-6x) = -60x^{2}$$

$$(x^2y)(xy)(x) = x^4y^2$$

$$(-4x)(-8y)(-2x^2) = -64x^3y$$

$$(5 x^2 y)(6 x^5 y^2) = 30 x^7 y^3$$

$$(-10 \times^2 y^3)(x^4 y^2)(9y) = -90 \times^6 y^6$$

$$(4x)(-3y)(2x)(2y) = -48x^2y^2$$

$$(-6x^5y^3z)(x^4z) = -6x^9y^3z^2$$

$$(x^2y)(x^2y)(x^2y)(x^2y) = x^8y^4$$

$$(2xy)(2xy)(2xy)(2xy) = 16x^4y^4$$

$$(4x^{4})(4x^{4})(4x^{4}) = 64x^{12}$$

$$(5x^5)(5x^5)(5x^5) = 125 \times 15$$

$$= 2/(2\pi)(2\pi) = 2^9 \times 9 = 5/2 \times 9$$

Then multiply the terms together. Power raised to a power, multiply.

$$(4x)^2 = (4x)(4x) = 16x^2$$

$$(6x)^2 = 6^2 \times^2 = 36 \times^2$$

$$(5w)^2 = 5^2 \omega^2 = 25\omega^2$$

$$(3x)^2 = 3^2 \times ^2 = 9 \times ^2$$

$$(7a)^2 = 7^2 a^2 = 49a^2$$

$$(10z)^2 = |0^2z^2 = |00z^2$$

$$(3x^2y^3)^2 = (3x^2y^3)(3x^2y^3) = 9x^4y^6$$

$$(8a^3b)^2 = 8a^3b^2 = 64a^6b^2$$

$$(10ab^4)^2 = 10^2 a^2 b^{4.2} = 100a^2 b^8$$

$$(6 \times yz)^2 = 6^2 \times^2 y^2 z^2 = 36 \times^2 y^2 z^2$$

$$(a^3b^4)^2 = a^{3\cdot 2}b^{4\cdot 2} = a^6b^8$$

$$(-4x^2)^3 = (-4x^2)(-4x^2)(-4x^2) = -64x^6$$

$$(3x^2)^3 = 3^3 \times^{2.3} = 27 \times^6$$

$$(5a^3)^3 = 5^3 a^{3\cdot 3} = 125a^9$$

$$(-2xy)^3 = (-2)^3 \times^3 y^3 = -8 \times^3 y^3$$

$$(a^2b^5)^3 = a^{2\cdot 3}b^{5\cdot 3} = a^6b^{15}$$

$$(2x^3)^3 = 2^3 \times 3^{3 \cdot 3} = 8 \times 9$$

$$(2x^3)^4 = 2^4 \times^{3.4} = 16 \times^{12}$$

$$(2x^3)^5 = 2^5 \times {}^{3.5} = 32 \times {}^{15}$$

$$(2x^3)^6 = 2^6 \times {}^{3.6} = 64 \times {}^{18}$$

$$(2 \times^2)^3 (5 \times)^2 = (2 \times^2)(2 \times^2)(2 \times^2)(5 \times)(5 \times) = 200 \times^8$$

$$(3x)^{4}(x^{2}y)^{3}=(3^{4}x^{4})(x^{2})^{3}=81x^{4+6}y^{3}=81x^{6}y^{3}$$

$$(2x^{2})^{3}(5x)^{2} = (2x^{2})(2x^{2})(2x^{2})(5x)(5x) = 200 x^{8}$$

$$(3x)^{4}(x^{2}y)^{3} = (3^{4}x^{4})(x^{2})^{3}y^{3} = 81x^{4+6}y^{3} = 81x^{6}y^{3}$$

$$(3x^{4})^{2}(2x)^{3} = 3^{2}\cdot 2^{3}x^{4+2}x^{3} = 9\cdot 8x^{8+3} = 72x^{11}$$

$$(a^{2}b)^{3}(ab^{3})^{2} = a^{2}\cdot 3b^{3}a^{2}b^{3} = a^{6}b^{3}a^{2}b^{6} = a^{6+2}b^{3+6} = a^{8}b^{9}$$

$$(a^2h)^3(ah^3)^2 = a^{2\cdot3}h^3 a^2 b^{3\cdot2} = a^6b^3a^2b^6 = a^{6+2}b^{3+6} = a^6$$

Finding. Powers with a Calculator

With a calculator we can compute a power of a number very easily. To compute 35 we could press these keys:

The calculator shows:

3

11
Y
$\boldsymbol{\Lambda}$



ı	_
ı	J.
ı	9
١.	







On many calculators we can do this even more quickly. Here's how:

When we enter

the calculator shows



which is 32

243

then press



the calculator shows

27

which is 33

and again press



the calculator shows

81

which is 34

and once more press

the calculator shows

243

which is 35

Notice that we press the | = | key one less time than the exponent.

Here's how we would find 26:

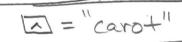






64

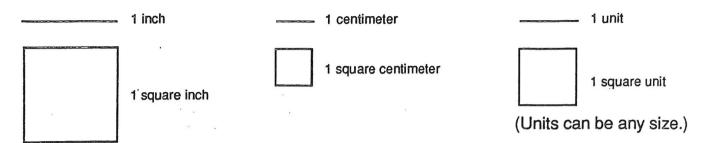
Do each problem using a calculator.



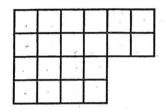


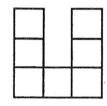
$$3^6 = 3 \land 6 = 729$$

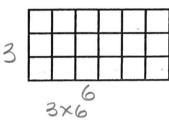
To measure length we use units like inches or centimeters. To measure area we need a unit which will cover a surface, so we have to use square units.

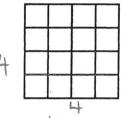


Find the area of each figure by counting the number of square units inside.









4×4

$$A = 20$$
 sq. units

$$A = 7$$
 sq. units

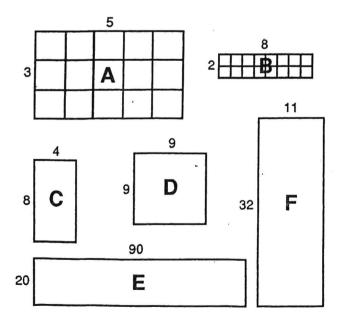
$$A = 18$$
 sq. units

$$A = 16$$
 sq. units

When the figure is a rectangle we can save time by just multiplying the length by the width to find the number of square units. That's what the **formula** below means.

$$A = l_{w}$$

Use this formula to find the area of each rectangle below.



Rectangle	l	W	A=lw
Α	5	3	A=5·3=15
В	8	2	A=8.2=16
С	4	8	A=4.8-32
D	9	9	A=9.9=81
E	90	20	A=90.20=1800
F	11	32	A=11.32=352

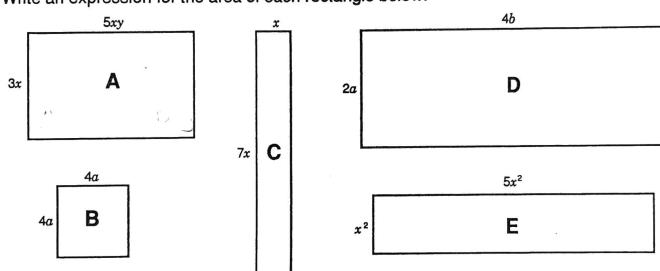
The length of each rectangle is two times its width. We can show a pattern for rectangles of this type by using a variable. If we use x to show the width, then the length is 2x.

Then we can use the formula A = lw to find an expression for the area.

$$A = \chi_W = (2\chi)(\chi) = 2\chi^2$$

x

Write an expression for the area of each rectangle below.



Rectangle	l	W	A=lw
Α	5ху	3x	$A = (5xy)(3x) = 15x^2y$
В	4a	49	A=(4a)(4a)=16a2
С	X	7x	$A = (x)(7x) = 7x^2$
D	46	2a	A= (4b)(2a)=8ab
E	5×2	X2	$A = (5x^2)(x^2) = 5x^4$

Terms that have equivalent variable parts are called like terms. Terms with variable parts that are not equivalent are called unlike terms.

2x, 3x and -5x are like terms.

 $^{-6}a^3$, a^3 , 5aaa and $32a^3$ are like terms.

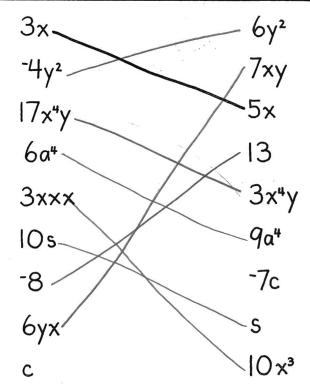
8, 1, -63 and -4 are like terms.

 $7x^3y^2$, x^3y^2 , 4xxxyy and $-6y^2x^3$ are like terms.

Look at each pair of terms and decide if they are like terms or unlike terms. Circle the right answer.

3x² and 4xx like unlike	2a³ and 5a³ unlike	4x and 7y like unlike
6x ⁴ and 2x ³ like unlike	3xy and 2yx	7c and 7
5 and 13	7x²y and 3yxx	4χ and 4χ

Match like terms.



ombining Like Terms

Do up to 31:

ook at these two expressions:

$$5x + 3x$$

Then we substitute different numbers for x, here's what we get:

	,		
χ	5x + 3x	X	8×
3	5(3)+3(3)=15+9=24	3	8(3)=24
10	5(10)+3(10)=50+30=80	10	8(10) = 80
-2	5(-2)+3(-2)=-10+-6=-16	, -2	8(-2) = -16
5	5(5)+3(5)=15+15=40	5	8(5)= 40

As you can see, we get the same answers each time. This happens because x + 3x and 8x are equivalent expressions. We can show that they are equivalent by using the Distributive Principle:

$$5x + 3x = (5+3)x = 8x$$

We can always use the Distributive Principle when we are adding like terms. Just add the number parts of the terms; the variable part stays the same. Here is another example:

$$2a^2 + 4a^2 = (2+4)a^2 = 6a^2$$

Of course, it's much easier to just write:

$$2a^2 + 4a^2 = 6a^2$$

Here are some expressions for you to simplify:

$$10xy + 7xy = 17xy$$

 $9x^4 + 5x^4 = 4x^4$
 $5s + 3s = -8s$

$$5x^2 + 7x^2 = -2x^2$$

$$6a^3 + 4a^3 = 10a^3$$

$$4x^2y + 3x^2y = 7x^2y$$

number parts. The variable part stays the same.)

$$3x^{2} + 9x^{2} = 12x^{2}$$
 $9w + 5w = 14w$
 $10a + 4a = 14a$
 $5yz + 5yz = 0yz = 0$
 $2y^{3} + 8y^{3} = 10y^{3}$
 $3 + 5 = -2$
 $13xy + 13xy = 26xy$
 $2x^{2} + 2x^{2} = 4x^{2}$
 $8a^{4} + 2a^{4} = 6a^{4}$
 $5a + 1a = 6a$
 $13m^{3} + 1m^{3} = 14m^{3}$

You already know that if x is any integer, then $1 \cdot x = x$. So when you see x in a problem you can change it to 1x. You can also change a to 1a, a^2 to $1a^2$, xy to 1xy, etc.

an change it to 1x. You can also change a to
$$1a$$
, a^2 to $1a^2$, xy to $1xy$, etc.

 $4x + x = 4x + 1x = 5x$
 $4x + 1x = 5x$
 $5x + 6x = 7x$
 $6x + 7x + 7x$
 $6x + 7x +$

e careful on these problems! Just add like terms.

$$\frac{4x + 8y + 3x}{1} = 7x + 8y$$

This terms

 $\frac{5y + 8y + 4z}{1} = 13y + 4z$
 $\frac{3 + 9b + 10}{1} = 9b + 13$
 $\frac{8x^2 + 2x^2 + 7x}{1} = 10x^2 + 7x$
 $\frac{6xy + 3xy + 3x}{1} = 9xy + 3x$
 $\frac{6xy + 3xy + 3x}{1} = 9xy + 3x$

$$7a + 5c + 4c = 7a + 9c$$

 $4x^{2} + 9 + 4x^{2} = 8x^{2} + 9$
 $x + 3y + 3x = 4x + 3y$
 $10x^{4} + 8x^{4} + 6x^{3} = 18x^{4} + 6x^{3}$
 $1xy + x + 1xy = 2xy + x$
 $1a + 1a + 5 = 2a + 5$
 $-4x^{2}y + -6 + -6x^{2}y = -10x^{2}y + -6$

$$6a + 7b + 5a + 7b = 11a + 14b$$

$$3x + 6y + 2y + 8x = 11x + 8y$$

$$9x^{2} + 10 + 4x^{2} + 7 = 13x^{2} + 17$$

$$4x + x + 3x + 8y = 8x + 8y$$

$$7x^{2}y + 8 + \frac{-5x^{2}y}{4} + 4 = 2x^{2}y + 12$$

$$5a + 3b + 4c + 2a = 7a + 3b + 4c$$

$$6x^{3} + 9x + 10x^{3} + 4x^{2} = 16x^{3} + 4x^{2} + 9x$$

$$8a^{2} + 4ab + 6a + 8a^{2} = 6a + 4ab$$

$$7a + 5b + c + 4a + \frac{-3}{2}b = 11a + 2b + c$$

$$(6xy + \frac{-8xy}{4}) + (5xy + \frac{-2xy}{4}) = xy$$

$$10x^{4} + \frac{-8x^{3}}{4} + 4x^{3} + \frac{-3x^{2}}{4} + 3x = 10x^{4} - 4x^{3} + -5x^{2} + 3x$$

$$4xy + \frac{-4xz}{4} + \frac{7xy}{4} + \frac{-11xy}{4} = -4xz$$

$$8x + 6 + 7x + \frac{-10}{4} + \frac{-5x}{4} + 8 = 10x + 14$$

Here are some expressions with subtraction. Every time you see a subtraction sign, you should add the opposite of the next term. Do all the figuring in your head. Just write down the answer.

$$3x - 7x = -4x$$

$$0.012\alpha + -2\alpha = 10\alpha$$

$$\frac{12a + 2a = 10a}{12a + 2a = 10a}$$

$$2xy + 7xy = -5xy$$

$$10x^2 + 6x^2 = 4x^2$$

$$5x + 3x + 11x = 5x + 3x + 11x = -3x$$

$$3a^2 + 6a^2 + 10a^2 = 7a^2$$

$$7xy + 5xy + 5xy = -3xy$$

$$6x^2y + 2x^2y + 10x^2y + 8x^2y = 2x^2y$$

$$9p + 3p + 9p + 3p = 0$$

$$a + 3a + a + 2a + 4a + 2a = 5a$$

$$9y + 13y = -4y$$

$$14a + 9a = 5a$$

$$3 + 8 = -5$$

$$x + 8x = -7x$$

$$4xy + xy = 3xy$$

$$\chi^2 + \chi^2 = 0$$

implify.

$$9x - 5x + x - 3x = 2x$$

$$5a + 2a - 4a + a = 4a$$

$$5xy - 2xy - 4xy - 3xy = 4xy$$

$$4x^{2} - 9x^{2} + x^{2} + 2x^{2} - 8x^{2} = -12x^{2}$$

$$10 + 3 + 2 + 4 = 1$$

$$8y + 2y + y + 4y + 5y + 10y = 6y$$

$$5(ab)^{2} + 6(ab)^{2} - 4(ab)^{2} + 3(ab)^{2} = 10(ab)^{2}$$

$$5x - 8x + 3x - 7x + 6x - 4x = -5x$$

$$8x^{4} + x^{4} - 6x^{4} - 4x^{4} - 5x^{4} = -6x^{4}$$

$$5ab - 3ab + (9ab - 6ab) + 7ab = 12ab$$

$$7c - 10c + 8c - c - c = 3c$$

$$4rt + (rt + rt) - 3rt + 5rt = 8rt$$

$$6z - 4z - z + 10z + 3z = 14z$$

$$x + x + x - x - x + x = 2x$$

$$5a^{2}bc^{3} - 7a^{2}bc^{3} + a^{2}bc^{3} + 2a^{2}bc^{3} = a^{2}bc^{3}$$

$$6 + 5 - 8 + 3 - 10 + 4 - 5 = -5$$

$$4m - 7m + 13m - 4m + 7m - 13m = 0$$

$$5y + y - 6y - y - y - y - y = -3y$$

$$g(3a+5b+7b-3a+2b)$$

 $3a+5b-7b=3a-2b$
 $8s-3s+4k=5s+4k$
 $10x+6y-5x=5x+6y$
 $10x-6y+5x=15x-6y$
 $16a+9-7a=9a+9$
 $a+2b-8a=-7a+2b$
 $2x-6y+7x+2y=9x-4y$
 $6s^2-3s^2+4t-6s^2=-3s^2+4t$
 $2b+4+3b+9=5b+13$
 $10-14xy+12xy+21=-2xy+31$
 $3x-7y+5x-y=6c-5-2c-7-8d=4c-8d-12$
 $2x+5y+3+7x+2y+7=9x+7y+10$
 $12+6p+3q-5p+7q-2=p+10q+10$
 $a^2+5a-3-7a+6a^2-4=7a^2-2a-7$
 $6x^2y+3xy^2-4x^2y-3xy^2+x^2y=3x^2y$
 $5+x-3+2x-x+7-8+x=3x+1$
 $x^2-7x+4+x^2+4x+6=2x^2-3x+10$
 $3x^2+5+x^2-9+x^2+16=5x^2+12$
 $5a-4b+2c-3b-6o+a=6a-7b-4c$

rimeters

e perimeter of a figure is the distance all the way around the outside of the figure. Write expression for the perimeter of each figure below.

ex	16221011 1	or the pe	inneter o	, odom ngare server
,	x + 2	t	Figure	Perimeter
	8		A	P = 3x + x + 2 + 3x + x + 2 = 8x + 4
3x	A	3x	В	P= x+6+ x+6+ x+1 = 3x+13
			С	P= 5a+5a+5a+5a=20a
			D	P = 3x + 2x + 2x + 3x + x + 2x = 13x
	x + 2		E	P = 4x + 8 + 6x + 6x = 16x + 8
	\wedge		F	P=3a-2b+8a+3a-2b+8a=22a-4b
x	+6	+ 6	G	P = l + w + l + w = 2l + 2w
	/ B \		Н	P=S+S+S+S=45
	$\frac{1}{x+1}$	i	1	P = a + b + c + d
			J	P=X2+5x+4+X2+2x2-3x+6=4x2+2x+10
	50		1	2x $3a-2b$
5a	C	;	5a	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
	56	<u> </u>]	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
			ı -	$x^2 + 5x + 4$
w			G	\mathbf{J}
			1	$2x^2-3x+6$
	8			a b c
s	H			d
	o			

Then write an expression for each other side and an expression for the perimeter. Simplify the expression for the perimeter if you can.

A triangle with two sides each 5 cm longer than the third side.

$$x+5 = x+x+5 + x+5 = 3x+10$$

A rectangular dog pen which is 5 times as long as it is wide.

The same dog pen after it is enlarged by adding 3 feet to the length.

$$P = 5x + 3 + x + 5x + 3 + x = 5x + 3$$

A helicopter pad which is a square.

$$x = x + x + x + x = 4x$$

A rectangular rug with a width that is 7 feet shorter than its length.

$$\times$$
 -7 $=$ \times \times -7 $=$ \times + \times -7 $=$ \times + \times -7 $=$ \times -14

Another rug which is twice as long as it is wide.

rder of Operations

p to now the simplifying problems you have done were only one step long. You just had to ook at the problem, think, and write down the answer.

this section are some simplifying problems that take more than one step — so you will ave to figure out what to do first. You can tell by using the following rule:

- 1. If there are parentheses, first do what is in them.
- 2. Then do all the multiplying, from left to right.
- 3. Finally, do the rest of the adding and subtracting, from left to right.

implify.

Simpiny.		10 (0 0)
5(4a + 2a) 5(6a) 30a	10y · 2y - 5y · 3y 20y² - 15y² 5y²	10x - (9x - 2x) 10x - 7x 3x
7(9x - 3x)	$5(3x^2) - (2x)(4x)$	14n + (3n - 5n)
7(6x)	$15x^2 - 8x^2$	14n + -2n
42x	$7x^2$	12n
(6y + 2y)3	-7(4a) - 6a(5)	3a - (7a - 2a)
(8y)3	-28a -30a	3a - 5a
24y	-58a	- 2a
4x (3x - 5x)	3x.8y + 2x.2y	$(7x^{2} - 5x^{2}) - x^{2}$
4x (-2x)	24xy + 4xy	$2x^{2} - x^{2}$
$-8x^{2}$	28xy	x^{2}
$x^{2}(4x + x)$ $x^{2}(3x)$ $3x^{3}$	$4a^2 \cdot a - 7a \cdot a^2$ $4a^3 - 7a^3$ $-3a^3$	(11b-12b)-5b (-b)-5b -6b

	,	· .
$2 \times (3 \times^{2} + 6 \times^{2})$ $2 \times (9 \times^{2})$ $18 \times^{3}$	$(3x^{2})(4x) - (5x)(2x^{2})$ $12x^{3} - 10x^{3}$ $2x^{3}$	5xy - (4xy - xy) 5xy - 3xy 2xy
5a³ (8a - 6a)	8xy + (3x)(4y)	$8x^{3} + (5x^{3} - 3x^{3})$
5a³ (2a)	8xy + (12xy)	$8x^{3} + (2x^{3})$
10a4	20xy	$10x^{3}$
$(4x^3 + 5x^3)(4x)$	$(5x^2)(4x^3) - 6x^5$	7a - (6a - a)
$(9x^3)(4x)$	$20x^5 - 6x^5$	7a - (5a)
$36x^4$	$14x^5$	2a
(5x-7x)(6y) (-2x)(6y) -12xy	$(3a)(7a^{4})+(4a^{3})(3a^{2})$ $21a^{5}+12a^{5}$ $33a^{5}$	$\frac{(4d^2 + 10d^2) - 6d^2}{14d^2 - 6d^2}$ $8d^2$
$(3x^2 + x^2)(5y^2 - 2y^2)$	$(5x^2)(2y^2) - (3xy)(6xy)$	(5xy-8xy)-4xy
$(4x^2)(3y^2)$	$10 \times ^2y^2 - 18x^2y^2$	-3xy-4xy
$12x^2y^2$	$-8x^2y^2$	-7xy

· - · · · · · · · · · · · ·

sing the Distributive Principle

he terms x and 3 are not like terms, so we cannot simplify 5(x + 3) by adding the terms in e parentheses. Instead, we use the Distributive Principle.

$$5(x+3) = 5x + 5.3 = 5x + 15$$

$$5(x-3) = 5x-5\cdot3 = 5x-15$$

Irite an equivalent expression using the Distributive Principle.

$$(x + 6) = 2x + 12$$

$$2(x-6) = 2x-12$$

$$3(2x + 4) = 6x + 12$$

$$8(x+2) = 8x + 6$$
 $8(x-2) = 8x - 6$

$$8(x-2) = 8x-16$$

$$11(5x + 2) = 55x + 22$$

$$6(x + 4) = 6x + 24$$

$$6(x-4) = 6x-24$$

$$6(x+4) = 6x+24$$
 $6(x-4) = 6x-24$ $-2(3x+1) = -6x-2$

$$(x+3)4 = 4x+12$$
 $(x-3)4 = 4x-12$

$$(x-3)4 = 4x-12$$

$$6(2x-3)=12x-18$$

$$x + 9)7 = 7x + 63$$

$$(x-9)7 = 1x-63$$

$$(x-9)7 = 1x-63$$
 $5(5x-2) = 25x-10$

$$3(x+1) = -3x-3$$

$$(x + 1)(-3) = -3 \times -3$$

$$-3(x+1) = -3x-3$$
 $(x+1)(-3) = -3x-3$ $(3x-10)(-5) = -15x+50$

$$5(x^2+6) = 5x^2+30$$

$$(x^2 - 6)5 = 5x^2 - 30$$

$$5(x^2+6) = 5x^2+30$$
 $(x^2-6)5 = 5x^2-30$ $(2x^2+1)(-3) = -6x^2-3$

Simplify.

$$8+3(x+2)$$

$$8 + 3x + 6$$

$$3x + 14$$

$$x + 4(x-6)$$

$$5(2x-3)+14$$

$$-2(x + 7) + 12x$$

$$x + 3(x - 4) + 2x$$

$$5x^2 + 3(x^2 - 1)$$

$$5x^2 + 3x^2 - 3$$

$$5y + (x-4)(-7)$$

$$\chi + 2(\chi + 1) + \chi^2$$

$$x^2 + 3x + 2$$

Find the number you get for each expression when you substitute 4 for x.

x + 5 4+5 9	x + 3 4+3	x + 10 4 + 10	x - 2 4-2 2	x - 6 4-6 -2	x - 4 4-4 0
3x 3(4) 12	5x 5(4) 20	9x 9(4) 36	-3x -3(4) -12	-5x -5(4) -20	1x 1(4) 4
3×+5 3(4)+5 12+5 17	2x + 3 2(4) + 3 8 + 3	8x+4 8(4)+4 32+4 36	3x-2 3(4)-2 12-2	5x-10 5(4)-10 20-10	2x-10 2(4)-10 8-10 -2
4(x+2) 4(4+2) 4(6) 24	5(x+3) 5(4+3) 5(7) 35	3(x+1) 3(4+1) 3(5) 15	7(x-1) 7(4-1) 7(3) 21	5(x-2) 5(4-2) 5(2)	3(x-7) 3(4-7) 3(-3) -9
x(x-2) 4(4-2) 4(2) 8	x(x+5) 4(4+5) 4(9) 36	x(x-7) 4(4-7) 4(-3) -12	x ² - 2x 4 ² - 2(4) 16 - 8 8	$x^{2} + 5x$ $(4)^{2} + 5(4)$ 16 + 20 36	$x^{2} - 7x$ $(4)^{2} - 7(4)$ $16 - 28$ -12
-x+2 -(4)+2 -2	-x+3 -(4)+3 -	-x+4 -(4)+4	-x+5 -(4)+5	-x-5 (4)+5	-x-4 -(4)+4 -8

nd the value of each expression when $\underline{a=5}$, $\underline{b=3}$ and $\underline{c=2}$.

The the value of each expression when $\underline{u}=0$, $\underline{v}=0$ and $\underline{v}=2$.							
a + b 5 + 3 8	a + 5+		b + c 3 + 2	a - b 5 - 3	b - 3 +	5	a - c 5-2 3
a b 5·3 15	ac (5)(1	2)	bc 3·2 6	a ² 5 ² 25	b 3 9		c ² 2 4
a+b+ 5+3+			-b-c 5+3+2	a - (b 5 - (3 -		5	(-(b+c) (-(3+2) (5-5)
a(b+c) 5(3+2) 5(5) 25		ab + ac $5.3 + 5.2$ $15 + 10$ $3(5+2)$ $3(7)$ 25		2)	3	ba + bc 3.5+3.2 15+6 21	
αbc 5.3.2 30		$a^{2} + b^{2}$ $5^{2} + 3^{2}$ $25 + 9$ 34		$a^{2} - c^{2}$ $5^{2} - 2^{2}$ $25 - 4$			$a^{2}c^{2}$ $5^{2} \cdot 2^{2}$ 100
(5+3)(9 (8)(9	$(a+b)(a+b)$ $a^2+2ab+b^2$ $(5+3)(5+3)$ $5^2+2(5)(3)+3^2$ $(8)(8)$ $25+30+9$ 64		(a+b)(a-b) (5+3)(5-3) (8)(2)		2	$a^2 - b^2$ $5^2 - 3^2$ $5 - 9$	

Practice Test

Finish each substitution table.

	4 _x	χ	x+6	×	
	4(5) = 20	5	5+6=11	5	
6	4:(6)=24	6	6+6=12	6	
-5	4(-5)=-20	⁻ 5	-5+6=1	- 5	
- 6	4(-6)=-24	-6	-6+6=0	-6	
0	4(0)=0	0	0+6=6	0	
					_

Use exponents to simplify each expression.

$$x \times x \times x \times x = x^{6}$$
 $(3y)(3y) = (3y)^{2}$
 $3aaaa = 3a^{4}$ $(mn)(mn)(mn) = (mn)^{3}$
 $-6xxyyyy = -6x^{2}y^{3}$ $6(ab)(ab) = 6(ab)^{2}$
 $2x \times x \times x = 2x^{4}$ $(2x)(2x)(2x)(2x) = (2x)^{4}$

Compute.

$$5^{2} = 25$$
 $(-3)^{2} = 9$ $8^{2} = 64$
 $2^{5} = 32$ $(-10)^{3} = -1000$

Simplify.

$$(8x^{3})(2x^{2}) = |6x^{5}|$$

$$(-5a)(3b) = -|5ab|$$

$$(-6xy)(-6xy) = 36x^{2}y^{2}$$

$$(x^{2}yz^{3})(xy^{4}) = x^{3}y^{5}z^{3}$$

$$(-3xy)(-2x^{2})(-4x^{2}y) = -24x^{5}y^{2}$$

$$(3x)(3x)(3x)(3x)(3x) = 81x^{4}$$

$$(2a)(3b)(5c) = 30abc$$

$$(xy^{2})(xyz)(xyz^{3}) = x^{3}y^{4}z^{4}$$

5 implify.

$$5a + 4b - 3a = 2a + 4b$$

$$6x^{2} - 8x^{2} - 5x = -2x^{2} - 5x$$

$$x + 7x + 5 = 6x + 5$$

$$6ab - 8ab + 4ab = 2ab$$

$$7x - 8y - 3x + 10y = 4x + 2y$$

$$x^{2} + 3x + 4 + x^{2} + 2x - 6 = 2x^{2} + 5x - 2$$

$$a + 7 - 4 + 3a + a - 2a = 3a + 3$$

$$6a + 4b - 3c - 6b = 6a - 2b - 3c$$

$$4x^{3} - 2x(10x^{2} - 3x^{2}) = 4x^{3} - 20x^{3} + 6x^{3} = 10x^{3}$$

$$(a + 4a) - 2(6a - 10a) = a + 4a - 12a + 20a = 13a$$

$$5x + 7(3(x^{2} - 7)) = 5x + 2 + 3x - 2 = 8x - 14$$

Do each problem in two steps. First write it out the long way. Then multiply terms.

$$(8a)^{2} = 8^{2}a^{2} = 64a^{2}$$

$$(3x)^{3} = 3^{3} \times^{3} = 27 \times^{3}$$

$$(2x^{2})^{4} = 2^{4} \times^{8} = 16 \times^{8}$$

$$(x^{2}y^{3})^{3} = x^{6}y^{9}$$

©1990 by Key Curriculum Project, Inc. Do not duplicate without permission.

Evaluate each expression for $\underline{a} = 2$ and $\underline{b} = 3$.

10a + 2b $10(2) + 2(-3)$ $20 + -6$	a+b 2+-3 -	a-b 2+(+3) 5
ab 2(-3) -6	3a + 2b 3(2) + 2(-3) 6 + -6	$a^{2} + b^{2}$ $2^{2} + (-3)^{2}$ $4 + 9$ 13