Polynomials

In Book 2 we said that a term is a very simple kind of expression where multiplication is the only operation. This book is about expressions that are made by adding and subtracting terms. These expressions are called **polynomials**.

$4y + 7$ is a polynomial in y.

$x^2 - 5x + 2$ is a polynomial in x.

$3a + 2b + 5c$ is a polynomial in a, b and c.

$3x$ is a polynomial in x.

$3a^4 + a^3 + 2a^2 + 6a - 7$ is a polynomial in a.

Most of the polynomials in this book are polynomials in the variable x.

We can list polynomials in x according to their **degree**. The degree of a polynomial in x is equal to the highest power of x that is in the polynomial.

Fourth Degree	*Third Degree*	*Second Degree*	*First Degree*	*Zero Degree*
$3x^4 + 5x^3 - 2x^2 + 5x - 4$	$6x^3 + 5x^2 + 4x - 2$	$x^2 + 5x + 6$	$3x + 4$	7
$6x^4 - 3x^2 + 7$	$x^3 + x$	$4x^2 - 9$	$5x$	-15
$4x^4 + x^3 - x + 1$	$3x^3 - 5x - 1$	$6x^2$	$x - 6$	5
$3x^4$	$4x^3$	$x^2 - 3x$	x	2

Tell the degree of each polynomial.

$4x^3 - 6x^2 + 4x - 2$ is a **third** degree polynomial in x.

$x^2 - 4$ is a second degree polynomial in x.

$x^2 - 5x + 2$ is a Second degree polynomial in x.

$6x^4$ is a fourth degree polynomial in x.

$x + 5$ is a first degree polynomial in x.

8 is a zero degree polynomial in x.

$5x^3 + 2x^2 - 4x$ is a third degree polynomial in x.

Adding Polynomials

Polynomials are like numbers in many ways. If we have two polynomials we can always add them together to get another polynomial. We just have to remember only to combine *like* terms. Here is an example:

$$(6x-7)+(4x+5)=10x-2$$

Here are some polynomials for you to add:

$(3x+5)+(4x+1)=$ $7x+6$

$(2x^2+3x+7)+(8x^2+3x-4)=$ $10x^2+6x+3$

$(x^2+5)+(x^2+4)=$ $2x^2+9$

$(5x-3)+(4x+7)+(2x-6)=$ $11x-2$

$(x^2+6x-5)+(x^2-8x-4)=$ $2x^2-2x-9$

$(3a+4b+c)+(5a-4b+2c)=$ $8a+3c$

$(3x+4)+(5x+2)+2x=$ $10x+6$

$(\cancel{5x^2}+\cancel{4x}\cancel{-7})+(\cancel{-5x^2}\cancel{-4x}\cancel{+7})=0$

$$\begin{array}{r} 3x^2+5x+2 \\ +\ \ 4x^2+3x+2 \\ \hline 7x^2+8x+4 \end{array}$$

$$\begin{array}{r} 5x^2-4x-8 \\ +\ \ 6x^2-9x+7 \\ \hline 11x^2-13x-1 \end{array}$$

$$\begin{array}{r} x^2+6x+9 \\ +\ \ x^2+4x+4 \\ \hline 2x^2+10x+13 \end{array}$$

$$\begin{array}{r} 3x-9 \\ +\ \ 2x^2+2x+2 \\ \hline 2x^2+5x-7 \end{array}$$

Opposites of Polynomials

There is another way that polynomials are like numbers. Every polynomial has an **opposite.** To find the opposite of a polynomial we just find the opposite of *each term* in the polynomial and then simplify.

For example, the opposite of $5x^2 - 3x + 7$

is $^{-}5x^2 - {}^{-}3x + {}^{-}7$

which simplifies to $^{-}5x^2 + 3x - 7$

You finish up the table below.

Polynomial	*Opposite Polynomial*
$3x^2 - 8x + 2$	$^{-}3x^2 + 8x - 2$
$6a + 7b + 4$	$-6a - 7b - 4$
$5x^2 - 2x - 9$	$-5x^2 + 2x + 9$
$x^2 - 16$	$-x^2 + 16$
$x^5 + x^4 + x^3 - x^2 + x - 1$	$-x^5 - x^4 - x^3 + x^2 - x + 1$
$^{-}7x - 8$	$7x + 8$
$x^2 + 5x - 14$	$-x^2 - 5x + 14$
$-x^2 - 5x + 14$	$x^2 + 5x - 14$

Let's see what happens when you add opposite polynomials together.

$$(\cancel{x^2} + \cancel{5x} - \cancel{14}) + (\cancel{-x^2} - \cancel{5x} + \cancel{14}) = 0$$

$$(\cancel{^{-}7x} - \cancel{8}) + (\cancel{7x} + \cancel{8}) = 0$$

$$(\cancel{5x^2} - \cancel{3x} + \cancel{7}) + (\cancel{^{-}5x^2} + \cancel{3x} - \cancel{7}) = 0$$

Subtracting Polynomials

Remember how we used to change subtraction problems to addition problems when we worked with integers? We always had to add the *opposite* of the *second number.* Well, polynomials work the same way — only this time we have to add the *opposite* of the *second polynomial.* Here is an example:

$$(5x^3 - 3x - 7) - (8x^3 + 6x - 2) =$$

First we have to change this to an addition problem. There are three terms in the second polynomial, so we have to be sure to change each of them.

$$(5x^3 - 3x - 7) + ({}^-8x^3 + {}^-6x + 2) =$$

Now we just have to write the answer:

$$(5x^3 - 3x - 7) + ({}^-8x^3 + {}^-6x + 2) = {}^-3x^3 - 9x - 5$$

You subtract these polynomials.

$$(3x^2 + 5x - 2) + (-7x^2 + 5x + {}^-4) = -4x^2 + 10x - 6$$

$$(6x^2 + 2x - 2) + (-x^2 + {}^-4x + 1) = 5x^2 - 2x - 1$$

$$(2y^2 - \cancel{y} + 3) + (-3y^2 + \cancel{y} + 4) = -y^2 + 7$$

$$(\cancel{y^2} + y \cancel{-6}) + (\cancel{-y^2} + {}^-5y \cancel{+6}) = -4y$$

$$(\cancel{4x^2} + 3x \cancel{+5}) + (\cancel{-4x^2} + 3x \cancel{+{}^-5}) = 6x$$

$$\begin{array}{rr} & 3x^2 + 5x - 2 \\ + & -2x^2 + 3x + {}^-7 \\ \hline & x^2 + 8x - 9 \end{array} \qquad \begin{array}{rr} & 5a^2 - 2a - 8 \\ + & -a^2 + 6a + {}^-3 \\ \hline & 4a^2 + 4a - 11 \end{array}$$

$$\begin{array}{rr} & \cancel{4x^2} - 3x + 2 \\ + & \cancel{-4x^2} + {}^-3x + {}^-6 \\ \hline & -6x - 4 \end{array} \qquad \begin{array}{rr} & 8x^2 - 8x - 3 \\ + & -4x + {}^-2 \\ \hline & 8x^2 - 12x - 5 \end{array}$$

In these problems you have to add *and* subtract polynomials. Change the signs on polynomials you are subtracting, but not on ones you are adding.

$(6x^2 - 4x + 7) + (2x^2 - 3x - 9) = 8x^2 - 7x - 2$

$(3x^2 + 5x - 1) + ({}^-4x^2 + 2x + {}^-4) = -x^2 + 7x - 5$

$(3x + 5) + (2x - 3) + (4x - 6) = 9x - 4$

$(a + b - c) + (a + b + 2c) + ({}^-a + {}^-b + {}^-c) = a + b$

$(2x^2 + x - 3) + (x^2 - 2x + 3) + ({}^-x^2 + {}^-x + {}^-3) + ({}^-x^2 + 3x + 12) = x^2 + x + 9$

$(x - y - z) + (x - y - z) + ({}^-x + y + z) + (x + y + z) = 2x$

$(3a^2 + 2b + 4) + ({}^-a^2 + {}^-b + 1) + ({}^-a^2 + {}^-2b + 1) + ({}^-a^2 + {}^-b + 2) = -2b + 8$

Solve each equation.

$8x - (5x - 4) = 25$
$8x + ({}^-5x + 4) = 25$
$3x + \cancel{4}^{-4} = 25^{-4}$
$3x = 21$
$x = 7$

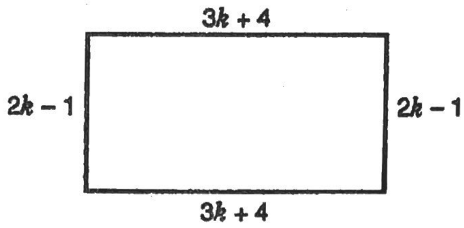
$6x + ({}^-4x + 5) = 13$
$6x - 4x + 5 = 13$
$2x + 5 = 13$
$-5 \quad -5$
$\frac{2x}{2} = \frac{8}{2}$
$\boxed{x = 4}$

$10x + ({}^-3x + {}^-6) = 8$
$10x - 3x - 6 = 8$
$7x - 6 = 8$
$+6 \quad +6$
$\frac{7x}{7} = \frac{14}{7}$
$\boxed{x = 2}$

$(6x + 9) + ({}^-2x + 5) = 38$
$6x + 9 - 2x + 5 = 38$
$4x + 14 = 38$
$-14 \quad -14$
$\frac{4x}{4} = \frac{24}{4}$
$\boxed{x = 6}$

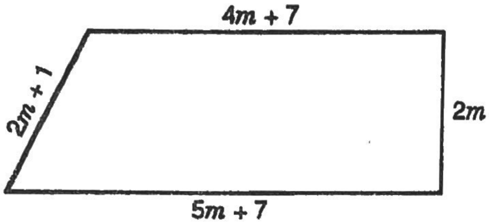
$(9x + 10) + ({}^-3x + {}^-2) = 74$
$9x + 10 - 3x - 2 = 74$
$6x + 8 = 74$
$-8 \quad -8$
$\frac{6x}{6} = \frac{66}{6}$
$\boxed{x = 11}$

Write a polynomial for the perimeter of each figure.

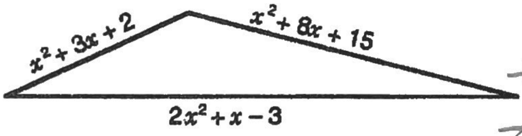


$$\begin{array}{r} 3k + 4 \\ 3k + 4 \\ 2k - 1 \\ 2k - 1 \\ \hline \end{array}$$

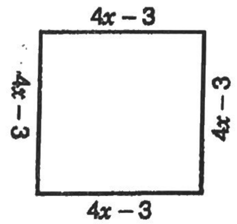
Answer: $P = 10k + 6$



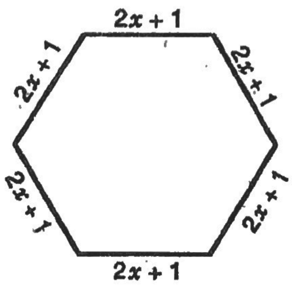
$$\begin{array}{r} 4m+7 \\ 2m \\ 5m+7 \\ +\ 2m+1 \\ \hline \boxed{13m+15} \end{array}$$



$$\begin{array}{r} x^2+3x+2 \\ x^2+8x+15 \\ +\ 2x^2+x-3 \\ \hline \boxed{4x^2+12x+14} \end{array}$$



$$\begin{array}{r} 4x-3 \\ 4x-3 \\ 4x-3 \\ +\ 4x-3 \\ \hline \boxed{16x-12} \end{array}$$



$6(2x+1)$

$\boxed{12x+6}$

The Distributive Principle

Remember the Distributive Principle from Book 1?

> Distributive Principle:
> If a, b and c are integers, then $a(b+c) = ab + ac$
> and $(b+c)a = ba + ca$.

This is the principle we use when we want to multiply a single term times a polynomial — we just multiply the single term times each term in the polynomial.

Here are some examples:

$$3(x+4) = 3x + 12$$

$$x(x+5) = x^2 + 5x$$

$$5x(2x+6) = 10x^2 + 30x$$

$$3x(x^2 - 5x + 2) = 3x^3 - 15x^2 + 6x$$

$$4x(2x^3 + 3x^2 - x + 6) = 8x^4 + 12x^3 - 4x^2 + 24x$$

To multiply a polynomial times a single term we use the second part of the Distributive Principle.

$$(x+4)6 = 6x + 24$$

$$(x-5)x = x^2 - 5x$$

$$(10x+3)5x = 50x^2 + 15x$$

$$(3x^2 + x - 7)2x = 6x^3 + 2x^2 - 14x$$

Use the Distributive Principle to do each multiplication problem below.

$3(2x-5) = 6x-15$

$5(6x-4) = 30x-20$

$(3a+4b)2 = 6a+8b$

$(x+5)10 = 10x+50$

$4(3x-y+5) = 12x-4y+20$

$5(3x-y+5) = 15x-5y+25$

$6(3x-y+5) = 18x-6y+30$

$(3x-y+5)7 = 21x-7y+35$

$-5(2x-4) = {}^-10x+20$

$-4(3y+5) = -12y-20$

$-3(2a-5b) = -6a+15b$

$(a+x)^-8 = -8a-8x$

$-4(3x^2-6x+2) = {}^-12x^2+24x-8$

$-5(3x^2-6x+2) = -15x^2+30x-10$

$-1(3x^2-6x+2) = -3x^2+6x-2$

$(3x^2-6x+2)(^-10) = -30x^2+60x-20$

$(2x-7)x = 2x^2-7x$

$(3y+5)y = 3y^2+5y$

$x(5-8y) = 5x-8xy$

$xy(3x+4y) = 3x^2y+4xy^2$

$y(2y^2+3x-4) = 2y^3+3xy-4y$

$x(2y^2+3x-4) = 2xy^2+3x^2-4x$

$(2y^2+3x-4)xy = 2xy^3+3x^2y-4xy$

$x^2(2y^2+3x-4) = 2x^2y^2+3x^3-4x^2$

$5x(4x-7) = 20x^2-35x$

$3a(4a+2) = 12a^2+6a$

$4x(x^2-5) = 4x^3-20x$

$(2x+1)2x^2 = 4x^3+2x^2$

$(3y-6)(^-5y) = -15y^2+30y$

$7x(3x+4y) = 21x^2+28xy$

$3a(4a-2b+c) = 12a^2-6ab+3ac$

$5y(3x+4y-8) = 15xy+20y^2-40y$

$2xy(3x+4y-8) = 6x^2y+8xy^2-16xy$

$(7x^2-5x-6)6x^2 = 42x^4-30x^3-36x^2$

$-4b(a-3b+c) = -4ab+12ab-4ac$

$7x^2y(3x^2y+2xy^2+x^3) =$

$21x^4y^2+14x^3y^3+7x^5y$

Solve each equation.

$5(x+3) = 35$

$5x + \cancel{15}^{-15} = 35^{-15}$

$5x = 20$

$x = 4$

$3(x+6) = 39$

$3x + 18 = 39$

$-18 -18$

$\frac{3x}{3} = \frac{21}{3}$

$\boxed{x = 7}$

$2(3x+15) = 18$

$6x + 30 = 18$

$-30 -30$

$\frac{6x}{6} = \frac{-12}{6}$

$\boxed{x = -2}$

$8(x-2) = 32$

$8x - 16 = 32$

$+16 +16$

$\frac{8x}{8} = \frac{48}{8}$

$\boxed{x = 6}$

$5(a+3) = 8a$

$5a + 15 = 8a$

$-5a -5a$

$\frac{15}{3} = \frac{3a}{3}$

$\boxed{a = 5}$

$7x = 4(x+6)$

$7x = 4x + 24$

$-4x -4x$

$\frac{3x}{3} = \frac{24}{3}$

$x = 8$

$4(x+5) = 3(x-6)$

$4x + 20 = 3x - 18$

$-3x \; -20 \quad -3x \quad -20$

$\boxed{x = -38}$

$3(2x-5) + 4 = 31$

$6x - 15 + 4 = 31$

$6x - 11 = 31$

$+11 +11$

$\frac{6x}{6} = \frac{42}{6}$

$\boxed{x = 7}$

$10 = 18 + 4(3x+7)$

$10 = 18 + 12x + 28$

$10 = 12x + 46$

$-46 -46$

$\frac{-36}{12} = \frac{12x}{12}$

$\boxed{x = -3}$

$3(3x+5) = 2(6x-3)$

$9x + 15 = 12x - 6$

$-9x -9x$

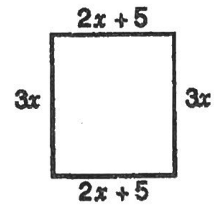
$15 = 3x - 6$

$+6 +6$

$\frac{21}{3} = \frac{3x}{3}$

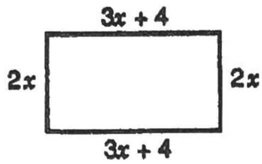
$\boxed{x = 7}$

Write a polynomial for the area of each rectangle.



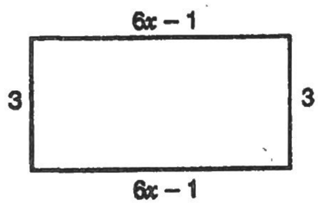
$A = 3x(2x + 5)$

$= 6x^2 + 15x$



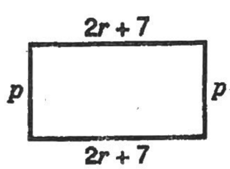
$A = 2x(3x + 4)$

$\boxed{A = 6x^2 + 8x}$



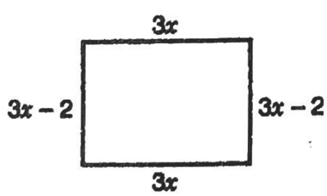
$A = 3(6x - 1)$

$\boxed{A = 18x - 3}$



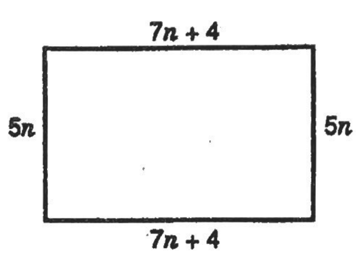
$A = p(2r + 7)$

$\boxed{A = 2pr + 7p}$



$A = 3x(3x - 2)$

$\boxed{A = 9x^2 - 6x}$



$A = 5n(7n + 4)$

$\boxed{A = 35n^2 + 20n}$

Factoring Out a Common Factor

Sometimes we need to break down a polynomial into a product of other polynomials. This is called **factoring**. If you could do the problems on the last three pages, then you won't have any trouble factoring single terms out of polynomials. Here is a polynomial that we will factor:

$$8x + 20 =$$

4 goes into both 8 and 20. So factor out 4:

$$8x + 20 = 4(\qquad)$$

Now it's easy to fill in the parentheses:

$$8x + 20 = 4(2x + 5)$$

Here's how to check your answer:

$$4(2x + 5) = 8x + 20$$

Here are some polynomials for you to factor.

$6a - 30 = 3(2a - 10)$	$5x - 5y + 10z = 5(x - y + 2z)$
$6a - 30 = 6(a - 5)$	$7x - 7y - 7z = 7(x - y - z)$
$6a - 30 = 2(3a - 15)$	$12a - 28b + 10c = 2(6a - 14b + 5c)$
$18x - 27 = 3(6x - 9)$	$2x^2 + 18x + 14 = 2(x^2 + 9x + 7)$
$18x - 27 = 9(2x - 3)$	$10x^2 + 15x - 20 = 5(2x^2 + 3x - 4)$
$4x - 32 = 4(x - 8)$	$6a - 18b + 12c = 6(a - 3b + 2c)$
$8y - 10 = 2(4y - 5)$	$9x + 12y + 15 = 3(3x + 4y + 5)$
$30a - 40 = 10(3a - 4)$	$8a + 12b + 4 = 4(2a + 3b + 1)$

Did you get the last problem? If not, see the next page.

Here's how Sandy and Terry did the last problem on page 11:

Sandy	*Terry*
$8a + 12b + 4 = 4(2a + 3b + 1)$	$8a + 12b + 4 = 4(2a + 3b)$

Here's what happened when they checked their answers:

Sandy	*Terry*
$4(2a + 3b + 1) = 8a + 12b + 4$ C	Where's the 4? $4(2a + 3b) = 8a + 12b$ X

Sandy's answer checked. Terry's didn't because Terry just factored out the 4 without leaving any term in its place to multiply by.

Factor each polynomial Sandy's way.

$15y + 5 = 5(3y+1)$

$7a + 7 = 7(a+1)$

$12n - 3 = 3(4n-1)$

$5 + 20x = 5(1+4x)$

$6x - 2 = 2(3x-1)$

$14n + 21p + 7 = 7(2n+3p+1)$

$9x - 18y + 3 = 3(3x-6y+1)$

$11 - 22a + 44b = 11(1-2a+4b)$

$5x^2 + 10x + 5 = 5(x^2+2x+1)$

$28x^2 + 28x + 7 = 7(4x^2+4x+1)$

Factor the biggest number you can out of each polynomial.

$24x + 28 = 4(6x+7)$

$10a + 25b = 5(2a+5b)$

$40n - 24 = 8(5n-3)$

$33p + 55 = 11(3p+5)$

$18x - 30y = 6(3x-5y)$

$100 + 40z = 20(5+2z)$

$56x^2 + 42 = 14(4x^2+3)$

$20x + 60y - 100z = 20(x+3y-5z)$

$14a - 12b + 6c = 2(7a-6b+3c)$

$24 + 48x + 42x^2 = 6(4+8x+7x^2)$

$50a - 20b + 30c = 10(5a-2b+3c)$

$6c^2 + 27c - 15 = 3(2c^2+9c-5)$

$12r + 36s - 60t = 12(r+3s-5t)$

$18x - 12y - 3 = 3(6x-4y-1)$

Any number that goes into all the terms of a polynomial is a **common factor** of those terms. Often we need to factor out the biggest possible number. This is called the **greatest common factor**. Factoring out the greatest common factor can be done in steps if you don't find it on the first try.

$$42a + 56 = 2(21a + 28)$$

2 goes into 42 and 56, so factor out 2.

$$= 2 \cdot 7(3a + 4)$$

7 goes into 21 and 28, so factor out 7.

$$= 14(3a + 4)$$

Factor out the greatest common factor. Do it in steps if you need to.

$16x - 48 =$ $16(x-3)$	$200x + 80y - 120z =$ $10(20x+8y-12z)$ $40(5x+2y-3z)$
$30x + 45 =$ $15(2x+3)$	$72a - 96b - 48c =$ $24(3a-4b-2c)$
$75x - 15 =$ $15(5x-1)$	$32x^2 + 40x + 160 =$ $8(4x^2+10x+20)$

Common factors can be variables as well as numbers.

$$x^2 + 8x = x\,x + 8\,x$$
$$= x(x+8)$$

x is a common factor.

$$2a^2 + a^3 = 2\,a\,a + a\,a\,a$$
$$= a^2(2+a)$$

$aa = a^2$, so a^2 is a common factor.

Here are some for you to try.

$x^2 + 3x = x(x+3)$	$a^3 + 5a^2 + 3a = a(a^2+5a+3)$
$5a^2 + 2a = a(5a+2)$	$2x^3 + x^2 - 8x = x(2x^2+x-8)$
$y^2 - 7y = y(y-7)$	$ab + 2b + b^2 = b(a+2+b)$
$12x - x^2 = x(12-x)$	$5x^2y + xy + 7y = y(5x^2+x+7)$
$3x^3 + 2x^2 = x^2(3x+2)$	$x^3 - 4x^2 + x = x(x^2-4x+1)$
$x^4 - 5x^2 = x^2(x^2-5)$	$ab + b^2 + 2bc = b(a+b+2c)$

Sometimes the terms of a polynomial have several common factors.

$$5x^4 + 15x^3 = 5 \cdot x^3 \cdot x + 5 \cdot 3 \cdot x^3$$
$$= 5x^3(x+3)$$

$5x^3$ is a common factor.

Here are some more polynomials for you to factor. Always factor out the biggest single term you can.

$5x^2 + 10x = 5x(x+2)$

$8a^2 - 2a = 2a(4a-1)$

$4y^5 + 3y^3 = y^3(4y^2+3)$

$x^2y^2 + x^3y = x^2y(y+x)$

$x^2y^2 + xy = xy(xy+1)$

$3x + 12x^2 = 3x(1+4x)$

$6x^3 - 9x^2 = 3x^2(2x-3)$

$4x^2 + 6xy = 2x(2x+3y)$

$10x^5 - 15x^4 = 5x^4(2x-3)$

$12a^2 - 18ab = 6a(2a-3b)$

$7x^3 - 7x^2 = 7x^2(x-1)$

$25x + 30xy = 5x(5+6y)$

$x^6 + x^5 - x^4 = x^4(x^2+x-1)$

$a^7 - a^5 - a^3 = a^3(a^4-a^2-1)$

$5a^5 + 3a^4 + 6a^3 = a^3(5a^2+3a+6)$

$6x^2y - xy^2 + 2x^2y^2 = xy($
$xy(6x-y+2xy)$

$x^3y + x^2y + x^2y^2 =$
$x^2y(x+1+y)$

$a^3b^3 + a^2b^2 + ab =$
$ab(a^2b^2+ab+1)$

$12a^3 - 9a^2 - 6a = 3a($
$3a(4a^2-3a-2)$

$10x^3 + 4x^2 + 6x =$
$2x(5x^2+2x+3)$

$8xy + 8y^2 - 8yz =$
$8y(x+y-z)$

$4x^5 - 4x^4 + 8x^3 =$
$4x^3(x^2-x+2)$

$63x^4 + 81x^3 - 72x^2 =$
$9x^2(7x^2+9x-8)$

$60a^2 + 30ab - 90ac =$
$30a(2a+b-3c)$

Monomials, Binomials and Trinomials

From now on we are going to call single terms **monomials**. Polynomials that have two terms will be called **binomials**; and polynomials with three terms will be called **trinomials**. See if you can add some more examples to the ones that are given below.

	Monomials	*Binomials*	*Trinomials*
	$3x$	$x + 5$	$x^2 + 7x + 10$
	y	$7a - 4$	$3x^2 - 9x + 6$
	$6x^2$	$3x^3 + 2x^2$	$x + y - z$
	$^{-}8$	$a + b$	$2x^4 + 8x^3 - x^2$
You write some.	$4x^7$	$2x - 5$	$6x^7 + x^4 - 9$
	$-9ab^2c$	$a^2 + 3a$	$a^2b + ab + b^2$

Multiplying and Factoring Polynomials

We already know how to multiply monomials (single terms) times other polynomials by using the Distributive Principle. Are you ready to multiply binomials times binomials? Well, here we go . . . All we have to remember is to multiply each term in the first binomial times each term in the second binomial. Here is an example:

$$(x + 5)(x + 3) =$$

First multiply x times each term in the second binomial.

$$(x + 5)(x + 3) = x^2 + 3x$$

Then multiply 5 times each term in the second binomial,

$$(x + 5)(x + 3) = x^2 + 3x + 5x + 15$$

Now simplify the answer by adding like terms.

$$\begin{aligned}(x + 5)(x + 3) &= x^2 + 3x + 5x + 15 \\ &= x^2 + 8x + 15\end{aligned}$$

Below are some problems for you to multiply out yourself.

$(x+3)(x+2) = x^2 + 2x + 3x + 6$

$= x^2 + 5x + 6$

$(x+5)(x+6) =$ $\boxed{x^2+11x+30}$

$\begin{array}{c|c|c} & x & 5 \\ \hline x & x^2 & 5x \\ \hline 6 & 6x & 30 \end{array}$

$(x+5)(x+2) =$ $\boxed{x^2+7x+10}$

$\begin{array}{c|c|c} & x & 5 \\ \hline x & x^2 & 5x \\ \hline 2 & 2x & 10 \end{array}$

$(x+1)(x+6) =$ $\boxed{x^2+7x+6}$

$\begin{array}{c|c|c} & x & 1 \\ \hline x & x^2 & x \\ \hline 6 & 6x & 6 \end{array}$

$(y+3)(y+5) =$ $\boxed{y^2+8y+15}$

$\begin{array}{c|c|c} & y & 3 \\ \hline y & y^2 & 3y \\ \hline 5 & 5y & 15 \end{array}$

$(a+8)(a+9) =$ $\boxed{a^2+17a+72}$

$\begin{array}{c|c|c} & a & 8 \\ \hline a & a^2 & 8a \\ \hline 9 & 9a & 72 \end{array}$

$(y+4)(y+10) =$ $\boxed{y^2+14y+40}$

$\begin{array}{c|c|c} & y & 4 \\ \hline y & y^2 & 4y \\ \hline 10 & 10y & 40 \end{array}$

$(z+3)(z+3) =$ $\boxed{z^2+6z+9}$

$\begin{array}{c|c|c} & z & 3 \\ \hline z & z^2 & 3z \\ \hline 3 & 3z & 9 \end{array}$

The problems below have negative terms, so be extra careful.

$(x-4)(x-5) = x^2 - 5x - 4x + 20$

$= x^2 - 9x + 20$

$(x-7)(x+3) = x^2 + 3x - 7x - 21$

$= x^2 - 4x - 21$

$(x-2)(x-4) =$ $\boxed{x^2-6x+8}$

$\begin{array}{c|c|c} & x & -2 \\ \hline x & x^2 & -2x \\ \hline -4 & -4x & 8 \end{array}$

$(x-5)(x+3) =$ $\boxed{x^2-2x-15}$

$\begin{array}{c|c|c} & x & -5 \\ \hline x & x^2 & -5x \\ \hline 3 & 3x & -15 \end{array}$

$(x-3)(x-5) =$ $\boxed{x^2-8x+15}$

$\begin{array}{c|c|c} & x & -3 \\ \hline x & x^2 & -3x \\ \hline -5 & -5x & 15 \end{array}$

$(x+2)(x-6) =$ $\boxed{x^2-4x-12}$

$\begin{array}{c|c|c} & x & 2 \\ \hline x & x^2 & 2x \\ \hline -6 & -6x & -12 \end{array}$

$(x-6)(x-6) =$ $\boxed{x^2-12x+36}$

$\begin{array}{c|c|c} & x & -6 \\ \hline x & x^2 & -6x \\ \hline -6 & -6x & 36 \end{array}$

$(x+8)(x-5) =$ $\boxed{x^2+3x-40}$

$\begin{array}{c|c|c} & x & 8 \\ \hline x & x^2 & 8x \\ \hline -5 & -5x & -40 \end{array}$

$(x-1)(x-8) =$ $\boxed{x^2-9x+8}$

$\begin{array}{c|c|c} & x & -1 \\ \hline x & x^2 & -x \\ \hline -8 & -8x & 8 \end{array}$

$(x-8)(x+5) =$ $\boxed{x^2-3x-40}$

$\begin{array}{c|c|c} & x & -8 \\ \hline x & x^2 & -8x \\ \hline 5 & 5x & -40 \end{array}$

Multiply.

$(x-3)(x-2) = \boxed{x^2-5x+6}$

	x	-3
x	x^2	-3x
-2	-2x	6

$(x+5)(x-8) = \boxed{x^2-3x-40}$

	x	5
x	x^2	5x
-8	-8x	-40

$(x-1)(x-1) = \boxed{x^2-2x+1}$

	x	-1
x	x	-x
-1	-x	1

$(x+4)(x-4) = \boxed{x^2-16}$

	x	4
x	x^2	$\cancel{4x}$
-4	$\cancel{-4x}$	-16

$(x-7)(x-2) = \boxed{x^2-9x+14}$

	x	-7
x	x^2	-7x
-2	-2x	14

$(x+1)(x-5) = \boxed{x^2-4x-5}$

	x	1
x	x^2	x
-5	-5x	-5

$(x-3)(x-4) = \boxed{x^2-7x+12}$

	x	-3
x	x^2	-3x
-4	-4x	12

$(x-3)(x+4) = \boxed{x^2+x-12}$

	x	-3
x	x^2	-3x
4	4x	-12

$(x+8)(x-3) = \boxed{x^2+5x-24}$

	x	8
x	x^2	8x
-3	-3x	-24

$(x-5)(x-2) = \boxed{x^2-7x+10}$

	x	-5
x	x^2	-5x
-2	-2x	10

$(x+5)(x-2) = \boxed{x^2+3x-10}$

	x	5
x	x^2	5x
-2	-2x	-10

$(x-6)(x+6) = \boxed{x^2-36}$

	x	-6
x	x^2	$\cancel{-6x}$
6	$\cancel{6x}$	-36

$(a-4)(a-6) = \boxed{a^2-10a+24}$

	a	-4
a	a^2	-4a
-6	-6a	24

$(y+3)(y-3) = \boxed{y^2-9}$

	y	3
y	y^2	$\cancel{3y}$
-3	$\cancel{-3y}$	-9

$(x-3)(x+5) = \boxed{x^2+2x-15}$

	x	-3
x	x^2	-3x
5	5x	-15

$(x+9)(x-2) = \boxed{x^2+7x-18}$

	x	9
x	x^2	9x
-2	-2x	-18

Multiply.

$(x+5)^2 = (x+5)(x+5)$
$= x^2+5x+5x+25$
$= x^2+10x+25$

$(x-5)^2 = (x-5)(x-5)$

$\boxed{x^2-10x+25}$

	x	-5
x	x^2	-5x
-5	-5x	25

$(x+4)^2 = (x+4)(x+4)$

$\boxed{x^2+8x+16}$

	x	4
x	x^2	4x
4	4x	16

$(x-7)^2 = (x-7)(x-7)$

$\boxed{x^2-14x+49}$

	x	-7
x	x^2	-7x
-7	-7x	49

$(x+10)^2 = (x+10)(x+10)$

$\boxed{x^2+20x+100}$

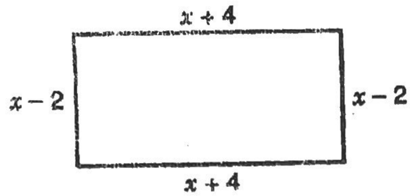
	x	10
x	x^2	10x
10	10x	100

$(x-1)^2 = (x-1)(x-1)$

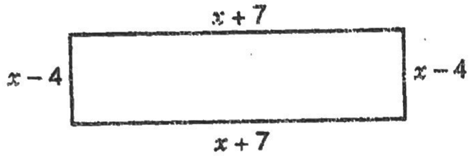
$\boxed{x^2-2x+1}$

	x	-1
x	x^2	-x
-1	-x	1

Write a polynomial for the area of each rectangle.



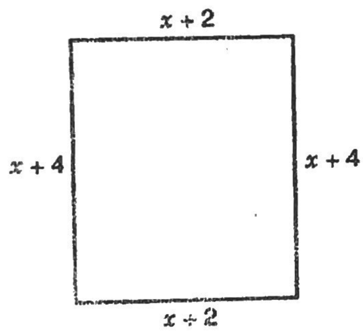
$A = (x-2)(x+4)$
$= x^2 + 4x - 2x - 8$
$= x^2 + 2x - 8$



$A = (x+7)(x-4)$

	x	7
x	x^2	$7x$
-4	$-4x$	-28

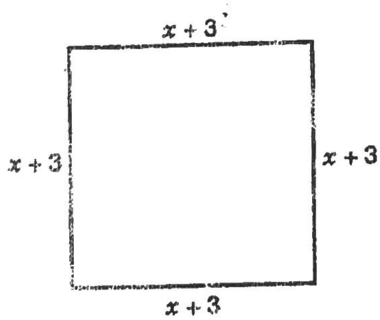
$\boxed{x^2 + 3x - 28}$



$A = (x+2)(x+4)$

	x	2
x	x^2	$2x$
4	$4x$	8

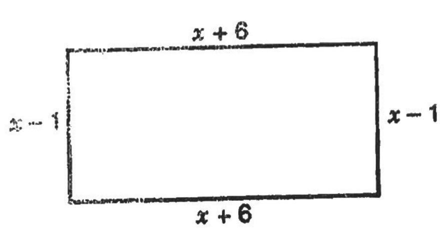
$\boxed{x^2 + 6x + 8}$



$A = (x+3)(x+3)$

	x	3
x	x^2	$3x$
3	$3x$	9

$\boxed{x^2 + 6x + 9}$



$A = (x+6)(x-1)$

	x	6
x	x^2	$6x$
-1	$-x$	-6

$\boxed{x^2 + 5x - 6}$

To do the next set of problems you will need a pencil and an eraser. You will need to make some guesses — and if your guess turns out to be wrong, you will need to erase it and guess again.

Each problem is a second degree trinomial in x. Your job is to factor the trinomial into a product of two binomials. Here is an example:

$$x^2 + 8x + 12 = (\quad\quad)(\quad\quad)$$

First factor the 12 as many different ways as you can.

3·4
1·12
2·6

$$x^2 + 8x + 12 = (\quad\quad)(\quad\quad)$$

Now test out the different combinations until you find one that works.

$(x+3)(x+4)$	$(x+1)(x+12)$	$(x+2)(x+6)$
$x^2+4x+3x+12$	$x^2+12x+1x+12$	$x^2+6x+2x+12$
$x^2+7x+12$	$x^2+13x+12$	$x^2+8x+12$ Yes!

As you can see, all three combinations gave us the correct first and last terms of the trinomial, but only the third combination gave us $8x$ for the middle term. So here is the answer:

$$x^2 + 8x + 12 = (x+2)(x+6)$$

Look at the problems below. Can you fill in each pair of blanks with the right numbers? Check each answer to be sure you are right.

3·4
2·6
1·12

What factors of 12 add up to 7?

$x^2 + 7x + 12 = (x + \underline{3})(x + \underline{4})$

$x^2 + 4x + 3x + 12$

$x^2 + 7x + 12$

4·5
2·10
1·20

What factors of 20 add up to 21?

$x^2 + 21x + 20 = (x + \underline{20})(x + \underline{1})$

4·5
2·10
1·20

$x^2 + 12x + 20 = (x + \underline{10})(x + \underline{2})$

4·6
3·8
2·12
1·24

$x^2 + 14x + 24 = (x + \underline{12})(x + \underline{2})$

4·5
2·10
1·20

$x^2 + 9x + 20 = (x + \underline{4})(x + \underline{5})$

4·6
3·8
2·12
1·24

$x^2 + 11x + 24 = (x + \underline{8})(x + \underline{3})$

Each of these problems is a second degree polynomial in x. Factor each one into a produ[ct] of two binomials. Check any answer you are not sure of.

$3\cdot6$, $2\cdot9$, $1\cdot18$

$x^2 + 9x + 18 = (x+3)(x+6)$

check: $x^2 + 6x + 3x + 18$

$x^2 + 9x + 18$

$x^2 + 11x + 18 = (x+9)(x+2)$

18, 9, 2, 11

$x^2 + 19x + 18 = (x+18)(x+1)$

18, 18, 1, 19

$x^2 + 7x + 10 = (x+5)(x+2)$

10, 5, 2, 7

$x^2 + 11x + 10 = (x+10)(x+1)$

10, 10, 1, 11

$x^2 + 4x + 4 = (x+2)(x+2)$

4, 2, 2, 4

$x^2 + 5x + 4 = (x+4)(x+1)$

4, 4, 1, 5

$x^2 + 2x + 1 = (x+1)(x+1)$

1, 1, 1, 2

$x^2 + 3x + 2 = (x+2)(x+1)$

2, 2, 1, 3

$6\cdot6$, $4\cdot9$, $3\cdot12$, $2\cdot18$, $1\cdot36$

$x^2 + 13x + 36 = (x+4)(x+9)$

36, 4, 9, 13

$x^2 + 20x + 36 = (x+18)(x+2)$

36, 18, 2, 20

$x^2 + 12x + 36 = (x+6)(x+6)$

36, 6, 6, 12

$x^2 + 37x + 36 = (x+1)(x+36)$

36, 37

$x^2 + 15x + 36 = (x+12)(x+3)$

36, 12, 3, 15

$x^2 + 17x + 30 = (x+15)(x+2)$

30, 15, 2, 17

$x^2 + 11x + 30 = (x+6)(x+5)$

30, 6, 5, 11

$x^2 + 13x + 30 = (x+10)(x+3)$

30, 10, 3, 13

$x^2 + 31x + 30 = (x+30)(x+1)$

30, 30, 1, 31

These polynomials have negative terms. If you factor them very carefully, you can make the numbers and the signs come out right every time.

$4 \cdot 4$, $2 \cdot 8$, $1 \cdot 16$ — What factors of 16 add to $^-10$?

$^-4 \cdot 7$, $4 \cdot {}^-7$, $^-2 \cdot 14$, $2 \cdot {}^-14$, $^-1 \cdot 28$, $1 \cdot {}^-28$ — What factors of $^-28$ add to $^-3$?

$x^2 - 10x + 16 = (x-2)(x-8)$

check: $x^2 - 8x - 2x + 16$

$x^2 - 10x + 16$

$x^2 - 3x - 28 = (x+4)(x-7)$

check: $x^2 + 4x - 7x - 28$

$x^2 - 3x - 28$

$x^2 - 8x + 16 = (x-4)(x-4)$

$x^2 + 3x - 28 = (x-7)(x+4)$

$x^2 - 17x + 16 = (x-16)(x-1)$

$x^2 - 2x - 24 = (x-6)(x+4)$

$x^2 - 8x + 15 = (x-5)(x-3)$

$x^2 + 2x - 24 = (x-4)(x+6)$

$x^2 - 16x + 15 = (x-15)(x-1)$

$x^2 + 8x - 20 = (x-2)(x+10)$

$x^2 - 17x + 72 = (x-9)(x-8)$

$x^2 - 8x - 20 = (x-10)(x+2)$

$x^2 - 16 = (x+4)(x-4)$

check: $x^2 - 4x + 4x - 16$

$x^2 - 16$

$x^2 - 4 = (x+2)(x-2)$

$x^2 - 25 = (x+5)(x-5)$

$x^2 - 1 = (x+1)(x-1)$

$x^2 - 81 = (x+9)(x-9)$

$x^2 - 100 = (x+10)(x-10)$

Factor these second degree polynomials. Don't forget to check your answers.

$x^2 + 14x + 45 = (x+9)(x+5)$

$a^2 + 17a + 72 = (a+9)(a+8)$

$x^2 - 4x - 45 = (x-9)(x+5)$

$a^2 - a - 72 = (a-9)(a+8)$

$x^2 + 18x + 45 = (x+15)(x+3)$

$a^2 + a - 72 = (a-8)(a+9)$

$x^2 - 12x - 45 = (x-15)(x+3)$

$x^2 - 5x + 4 = (x-4)(x-1)$

$x^2 - 14x + 45 = (x-9)(x-5)$

$p^2 + 3p - 4 = (x-1)(x+4)$

$x^2 + 12x - 45 = (x-3)(x+15)$

$x^2 + 4x - 12 = (x-2)(x+6)$

$x^2 + 4x - 45 = (x-5)(x+9)$

$x^2 - 4x + 4 = (x-2)(x-2)$

$x^2 - 18x + 45 = (x-15)(x-3)$

$s^2 - 10s + 25 = (s-5)(s-5)$

$x^2 + 46x + 45 = (x+45)(x+1)$

$y^2 - 6y + 9 = (y-3)(y-3)$

$x^2 - 44x - 45 = (x-45)(x+1)$

$a^2 + 2a + 1 = (a+1)(a+1)$

$x^2 + 44x - 45 = (x+45)(x-1)$

$m^2 + 4m + 3 = (m+3)(m+1)$

$x^2 - 46x + 45 = (x-45)(x-1)$

$c^2 - c - 2 = (c+1)(c-2)$

Here are some more second degree polynomials to factor.

$b^2 + 4b - 60 = (b-6)(b+10)$

$a^2 - 3a - 28 = (a-7)(a+4)$

$m^2 - 2m - 63 = (m-9)(m+7)$

$x^2 - 7x - 18 = (x-9)(x+2)$

$m^2 + m - 30 = (m-5)(m+6)$

$a^2 - a - 20 = (a-5)(a+4)$

$a^2 + a - 20 = (a-4)(a+5)$

$z^2 + z - 12 = (z-3)(z+4)$

$x^2 + x - 42 = (x-6)(x+7)$

$x^2 - 2x - 15 = (x-5)(x+3)$

$x^2 - 2x - 3 = (x-3)(x+1)$

$x^2 + 2x - 3 = (x-1)(x+3)$

$r^2 - 2r - 63 = (r-9)(r+7)$

$x^2 - 8x + 12 = (x-6)(x-2)$

$x^2 - 9x + 8 = (x-1)(x-8)$

$s^2 + 13s + 42 = (s+7)(s+6)$

$x^2 - 14x + 48 = (x-8)(x-6)$

$x^2 - 9 = (x+3)(x-3)$

$a^2 - 36 = (a+6)(a-6)$

$y^2 - 64 = (y-8)(y+8)$

$x^2 - 1 = (x+1)(x-1)$

$x^2 - 49 = (x-7)(x+7)$

$x^2 - 16 = (x+4)(x-4)$

$m^2 - 144 = (m+12)(m-12)$

In each of these problems you have to multiply a *monomial* times a *binomial* times a *binomial*. It's easiest if you multiply the binomials together first.

$3(x-7)(x+2) = 3(x^2+2x-7x-14)$
$= 3(x^2-5x-14)$
$= 3x^2-15x-42$

$2(x+3)(x-3) =$
$2(x^2-9)$
$2x^2-18$

$4(x-3)(x+5) =$
$4(x^2-3x+5x-15)$
$4(x^2-2x-15)$
$4x^2-8x-60$

$a(x+1)(x+4) =$
$a(x^2+x+4x+4)$
$ax^2+5ax+4a$

$6(x+5)(x+4) =$
$6(x^2+5x+4x+20)$
$6(x^2+9x+20)$
$6x^2+54x+120$

$x(x-5)(x-4) =$
$x(x^2-5x-4x+20)$
x^3-9x^2+20x

Below are some trinomials for you to factor. First factor out the biggest monomial you ca
Then factor the trinomial that's left into a binomial times a binomial.

$6x^2+18x-60 = 6(x^2+3x-10)$
$= 6(x-2)(x+5)$

$x^3+8x^2-20x = x(x^2+8x-20)$
$= x(x+10)(x-2)$

$2x^2+10x-28 = 2(x^2+5x-14)$
$2(x-2)(x+7)$

$x^3+7x^2+10x =$
$x(x^2+7x+10)$
$x(x+5)(x+2)$

$5x^2+35x+60 =$
$5(x^2+7x+12)$
$5(x+4)(x+3)$

$a^3-7a^2+6a =$
$a(a^2-7a+6)$
$a(a-6)(a-1)$

Multiply each pair of binomials. Remember to multiply each term in the first binomial times each term in the second binomial.

$$(2x-3)(5x+4) = 10x^2 + 8x - 15x - 12$$
$$= 10x^2 - 7x - 12$$

$(3x-1)(4x+3) =$ $\boxed{12x^2+5x-3}$

	$3x$	-1
$4x$	$12x^2$	$-4x$
3	$9x$	-3

$(6x+1)(2x+3) =$ $\boxed{12x^2+20x+3}$

	$6x$	1
$2x$	$12x^2$	$2x$
3	$18x$	3

$(5x+3)(3x+4) =$ $\boxed{15x^2+29x+12}$

	$5x$	3
$3x$	$15x^2$	$9x$
4	$20x$	12

$(2x+5)(x-3) =$ $\boxed{2x^2-x-15}$

	$2x$	5
x	$2x^2$	$5x$
-3	$-6x$	-15

$(4x-1)(3x-1) =$ $\boxed{12x^2-7x+1}$

	$4x$	-1
$3x$	$12x^2$	$-3x$
-1	$-4x$	1

$(3x-5)(2x-5) =$ $\boxed{6x^2-25x+25}$

	$3x$	-5
$2x$	$6x^2$	$-10x$
-5	$-15x$	25

$(4x-3)(x+3) =$ $\boxed{4x^2+9x-9}$

	$4x$	-3
x	$4x^2$	$-3x$
3	$12x$	-9

$(3x+4)(3x-4) =$ $\boxed{9x^2-16}$

	$3x$	4
$3x$	$9x^2$	$\cancel{12x}$
-4	$\cancel{-12x}$	-16

$$(3x-4)^2 = (3x-4)(3x-4)$$
$$= 9x^2 - 12x - 12x + 16$$
$$= 9x^2 - 24x + 16$$

$(5x+2)^2 = (5x+2)(5x+2)$ $\boxed{25x^2+20x+4}$

	$5x$	2
$5x$	$25x^2$	$10x$
2	$10x$	4

$(2x-3)^2 =$ $\boxed{4x^2-12x+9}$

	$2x$	-3
$2x$	$4x^2$	$-6x$
-3	$-6x$	9

$(4x+1)^2 =$ $\boxed{16x^2+8x+1}$

	$4x$	1
$4x$	$16x^2$	$4x$
1	$4x$	1

$(3x-7)^2 =$ $\boxed{9x^2-42x+49}$

	$3x$	-7
$3x$	$9x^2$	$-21x$
-7	$-21x$	49

The Zero Product Rule

Write all the pairs of integer factors you can find for each of the following numbers. Then write the number of pairs you found.

16	14	19	-50	0
2 and 8 -2 and -8 4 and 4 -4 and -4 16 and 1 -16 and -1	2 · 7 -2 · -7 14 · 1 -14 · -1	19 · 1 -19 · -1	-50 · 1 -1 · 50 25 · -2 -25 · 2 5 · -10 -5 · 10	0 · 0 1 · 0 2 · 0 3 · 0
6 pairs	4 pairs	2 pairs	6 pairs	∞ pairs

If you tried to write *all* possible pairs of factors for 0, you wouldn't be finished yet. The number of pairs is unlimited! Any number will do for one factor as long as the other factor is 0. This leads us to the Zero Product Rule:

> If a product is 0, then at least one of the factors must be 0.
> Or, in algebraic terms: if $a \cdot b = 0$,
> then $a = 0$ or $b = 0$.

We can use this rule to solve equations like $(x-3)(x+2)=0$. All we need to do is find numbers which make either $x-3$ or $x+2$ equal to 0. There are two solutions. Can you see what they are? Finish the table to show that the solutions are 3 and -2.

x	$x-3$	$x+2$	$(x-3)(x+2)$
3	3-3	3+2	(3-3)(3+2)
-2	-2-3	-2+2	(-2-3)(-2+2)

Solve each equation using the Zero Product Rule.

$(x-7)(x-2)=0$ $x-7=0$ or $x-2=0$ $x=7$ or $x=2$	$(x-6)(x-1)=0$ $x-6=0$ $x-1=0$ $+6$ $+6$ $+1$ $+1$ $\boxed{x=6}$ $\boxed{x=1}$	$(x+3)(x-3)=0$ $x+3=0$ $x-3=0$ $\boxed{x=-3}$ $\boxed{x=3}$
$x(x-4)=0$ $x=0$ $x-4=0$ $\boxed{x=0}$ $\boxed{x=4}$	$3x(x-5)=0$ $\frac{3x}{3}=\frac{0}{3}$ $x-5=0$ $\boxed{x=0}$ $\boxed{x=5}$	$5x(x+8)=0$ $5x=0$ $x+8=0$ $\boxed{x=0}$ $\boxed{x=-8}$
$(x-2)(x-9)=0$ $x-2=0$ $x-9=0$ $\boxed{x=2}$ $\boxed{x=9}$	$(x-3)(x+6)=0$ $x-3=0$ $x+6=0$ $\boxed{x=3}$ $\boxed{x=-6}$	$(x+1)(x+5)=0$ $x+1=0$ $x+5=0$ $\boxed{x=-1}$ $\boxed{x=-5}$

Quadratic Equations

Equations like $(x-3)(x+2)=0$ are called **quadratic equations.** A quadratic equation can always be written with a second degree polynomial on one side and a 0 on the other side. $(x-3)(x+2)=0$ is equivalent to $x^2-x-6=0$, so it is a quadratic equation.

If we can factor the polynomial in a quadratic equation, we can solve it by using the Zero Product Rule. Here is an example:

$$x^2-8x-20=0$$

First we factor: $(x-10)(x+2)=0$

Then we use the Zero Product Rule: $x-10=0$ or $x+2=0$

Then we find the solutions: $x=10$ or $x={}^-2$

To check the solutions, we try them in the original equation.

$$(10)^2-8(10)-20 = 100-80-20=0$$

$$({}^-2)^2-8({}^-2)-20 = 4+16-20=0$$

Solve each equation by factoring and using the Zero Product Rule.

$x^2-7x+10=0$ $(x-5)(x-2)=0$ $x-5=0$ $x-2=0$ $\boxed{x=5}$ $\boxed{x=2}$	$x^2-5x-24=0$ $(x-8)(x+3)=0$ $x-8=0$ $x+3=0$ $\boxed{x=8}$ $\boxed{x=-3}$	$x^2-10x+9=0$ $(x-9)(x-1)=0$ $x-9=0$ $x-1=0$ $\boxed{x=9}$ $\boxed{x=1}$
$x^2+11x+30=0$ $(x+6)(x+5)=0$ $x+6=0$ $x+5=0$ $\boxed{x=-6}$ $\boxed{x=-5}$	$0=x^2+7x-18$ $0=(x-2)(x+9)$ $x-2=0$ $x+9=0$ $\boxed{x=2}$ $\boxed{x=-9}$	$0=x^2+15x+44$ $0=(x+11)(x+4)$ $x+11=0$ $x+4=0$ $\boxed{x=-11}$ $\boxed{x=-4}$
$x^2-49=0$ $(x+7)(x-7)=0$ $x+7=0$ $x-7=0$ $\boxed{x=-7 \quad x=7}$	$0=x^2-25$ $0=(x+5)(x-5)$ $x+5=0$ $x-5=0$ $\boxed{x=-5 \quad x=5}$	$x^2-100=0$ $(x+10)(x-10)=0$ $\boxed{x=-10 \quad x=10}$
$x^2-6x=0$ $x(x-6)=0$ $\boxed{x=0}$ $x-6=0$ $\boxed{x=6}$	$x^2+4x=0$ $x(x+4)=0$ $\boxed{x=0}$ $x+4=0$ $\boxed{x=-4}$	$x^2+4x-5=0$ $(x-1)(x+5)=0$ $x-1=0$ $x+5=0$ $\boxed{x=1}$ $\boxed{x=-5}$

First use the Addition Principle to make one side equal to 0. Then solve the equation by factoring.

To get a 0 here I need to add -2.

$$x^2 - 5x + 8 = 2$$
$$\quad -2 \quad -2$$
$$x^2 - 5x + 6 = 0$$
$$(x-3)(x-2) = 0$$
$$x - 3 = 0 \quad x - 2 = 0$$
$$x = 3 \quad x = 2$$

$$x^2 - 5x - 10 = 4$$
$$\quad -4 \quad -4$$
$$x^2 - 5x - 14 = 0$$
$$(x-7)(x+2) = 0$$
$$x - 7 = 0 \quad x + 2 = 0$$
$$x = 7 \quad x = -2$$

$$x^2 + 48 = 49$$
$$\quad -49 \quad -49$$
$$x^2 - 1 = 0$$
$$(x+1)(x-1) = 0$$
$$x = -1 \quad x = 1$$

$$x^2 - 6 = 19$$
$$\quad -19 \quad -19$$
$$x^2 - 25 = 0$$
$$(x+5)(x-5) = 0$$
$$x = -5 \quad x = 5$$

$$x^2 + 7x = 8$$
$$\quad -8 \quad -8$$
$$x^2 + 7x - 8 = 0$$
$$(x-1)(x+8) = 0$$
$$x - 1 = 0 \quad x + 8 = 0$$
$$x = 1 \quad x = -8$$

$$x^2 - 7x = -12$$
$$\quad +12 \quad +12$$
$$x^2 - 7x + 12 = 0$$
$$(x-3)(x-4) = 0$$
$$x - 3 = 0 \quad x - 4 = 0$$
$$x = 3 \quad x = 4$$

$$x^2 + 7x + 1 = 1$$
$$\quad -1 \quad -1$$
$$x^2 + 7x = 0$$
$$x(x+7) = 0$$
$$x + 7 = 0$$
$$x = 0 \quad x = -7$$

$$x^2 - 10x + 5 = 29$$
$$\quad -29 \quad -29$$
$$x^2 - 10x - 24 = 0$$
$$(x-12)(x+2) = 0$$
$$x - 12 = 0 \quad x + 2 = 0$$
$$x = 12 \quad x = -2$$

$$x^2 - 5x + 11 = 11$$
$$\quad -11 \quad -11$$
$$x^2 - 5x = 0$$
$$x(x-5) = 0$$
$$x - 5 = 0$$
$$x = 0 \quad x = 5$$

$$x^2 + 16x + 42 = 3$$
$$\quad -3 \quad -3$$
$$x^2 + 16x + 39 = 0$$
$$(x+13)(x+3) = 0$$
$$x + 13 = 0 \quad x + 3 = 0$$
$$x = -13 \quad x = -3$$

$$x^2 - 10x = -2x$$
$$\quad +2x \quad +2x$$
$$x^2 - 8x = 0$$
$$x(x-8) = 0$$
$$x - 8 = 0$$
$$x = 0 \quad x = 8$$

$$x^2 + 3x - 72 = 4x$$
$$\quad -4x \quad -4x$$
$$x^2 - x - 72 = 0$$
$$(x-9)(x+8) = 0$$
$$x - 9 = 0 \quad x + 8 = 0$$
$$x = 9 \quad x = -8$$

$$x^2 + 36 = 12x$$
$$\quad -12x \quad -12x$$
$$x^2 - 12x + 36 = 0$$
$$(x-6)(x-6) = 0$$
$$x - 6 = 0$$
$$x = 6$$

$$3x^2 - 3x - 8 = 2x^2 + 2$$
$$-2x^2 \quad -2 \quad -2x^2 \quad -2$$
$$x^2 - 3x - 10 = 0$$
$$(x-5)(x+2) = 0$$
$$x - 5 = 0 \quad x + 2 = 0$$
$$x = 5 \quad x = -2$$

$$x^2 + 5x + 5 = x + 1$$
$$\quad -x \quad -1 \quad -x \quad -1$$
$$x^2 + 4x + 4 = 0$$
$$(x+2)(x+2) = 0$$
$$x + 2 = 0$$
$$x = -2$$