

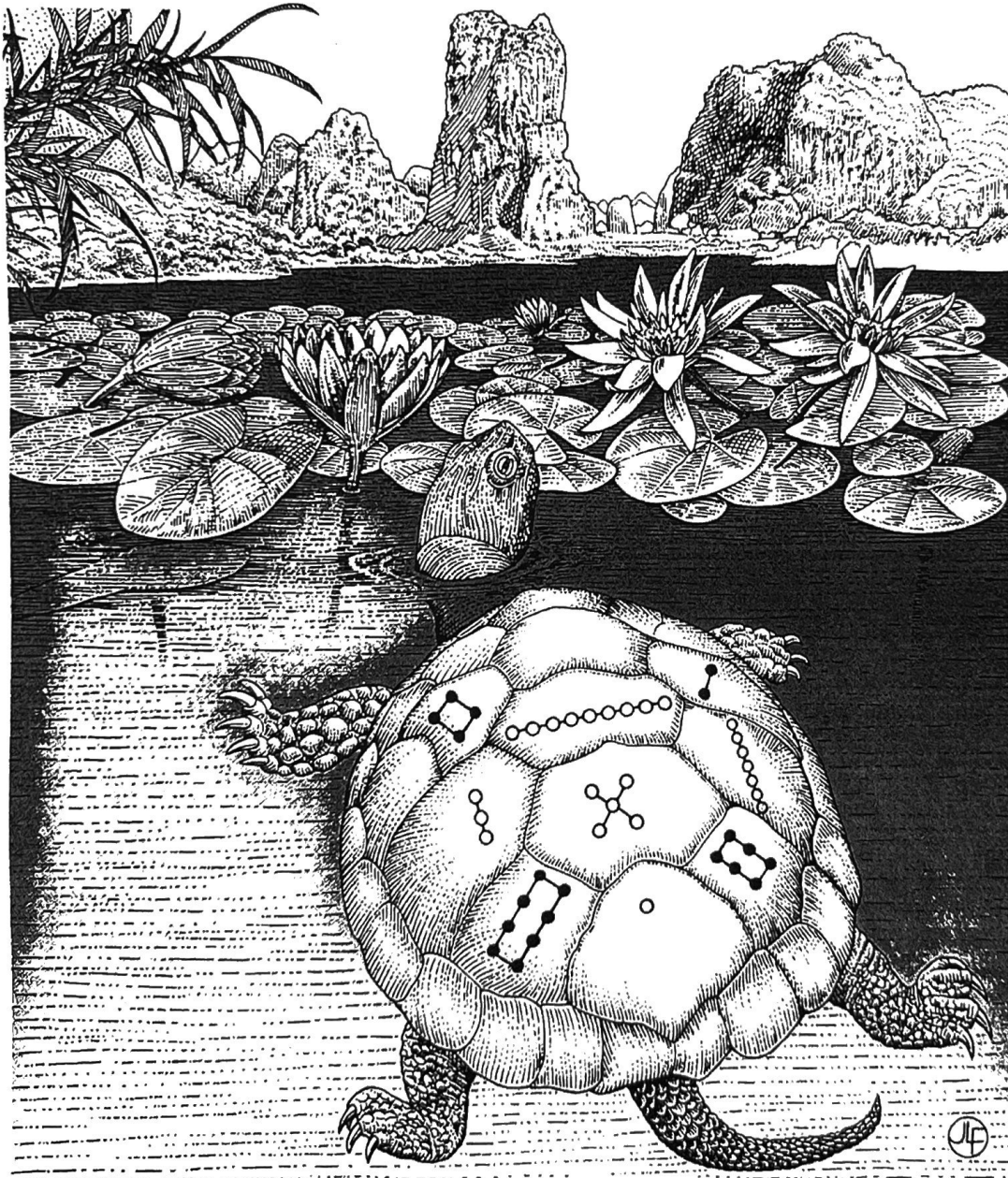
Key to

Algebra[®]

5
Student
Workbook

Solutions

Rational Numbers



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Name

Class

Rational Numbers

In Books 1 to 4 we worked with integers (positive and negative whole numbers and 0). We had no trouble adding, subtracting and multiplying integers, but when we got to division we ran into difficulties. Division problems like $9 \div 0$ have no answer, because you can never divide by 0. Other problems, like $10 \div 3$, do not have answers which are integers.

To solve problems like $10 \div 3$ we need a new class of numbers called **rational numbers**. Rational numbers are numbers which can be written as fractions. The **numerator** (top number) and **denominator** (bottom number) of a fraction must be integers and the denominator may not be 0.

$$\begin{array}{cccccccc} \frac{3}{4} & \frac{1}{7} & \frac{-5}{2} & \frac{0}{8} & \frac{-6}{11} & \frac{-9}{1} & \frac{-7}{3} & \leftarrow \text{numerators} \\ & & & & & & & \leftarrow \text{denominators} \end{array}$$

Every integer is a rational number because it can be written as a fraction with a denominator of 1. Rewrite each integer as a fraction.

$$8 = \frac{8}{1}$$
$$4 = \frac{4}{1}$$

$$-3 = \frac{-3}{1}$$
$$-21 = \frac{-21}{1}$$

$$0 = \frac{0}{1}$$
$$25 = \frac{25}{1}$$

$$-15 = \frac{-15}{1}$$
$$6 = \frac{6}{1}$$

Every mixed number is a rational number because it can be written as a fraction. Rewrite each mixed number as a fraction.

$2 \cdot 8 = 16$. 2 is 16 eighths and there are 3 more eighths.

$$2\frac{3}{8} = \frac{16+3}{8} = \frac{19}{8}$$

$$3\frac{2}{5} = \frac{15+2}{5} = \frac{17}{5}$$

$$1\frac{3}{4} = \frac{4+3}{4} = \frac{7}{4}$$

$$5\frac{1}{2} = \frac{10+1}{2} = \frac{11}{2}$$

$$4\frac{5}{6} = \frac{24+5}{6} = \frac{29}{6}$$

$$10\frac{2}{3} = \frac{30+2}{3} = \frac{32}{3}$$

$$7\frac{1}{7} = \frac{49+1}{7} = \frac{50}{7}$$

$$3\frac{3}{7} = \frac{21+3}{7} = \frac{24}{7}$$

Every decimal is also a rational number (unless it goes on forever without repeating). A **terminating decimal** (one that comes to an end) is a rational number because it equals a fraction with a denominator of 10 or 100 or 1000, etc. Rewrite each terminating decimal as a fraction.

$$0.6 = \frac{6}{10}$$

$$0.06 = \frac{6}{100}$$

$$0.006 = \frac{6}{1000}$$

$$0.056 = \frac{56}{1000}$$

$$0.9 = \frac{9}{10}$$

$$0.09 = \frac{9}{100}$$

$$0.19 = \frac{19}{100}$$

$$0.119 = \frac{119}{1000}$$

$$1.3 = 1\frac{3}{10} = \frac{13}{10}$$

$$2.7 = 2\frac{7}{10} = \frac{27}{10}$$

$$5.01 = 5\frac{1}{100} = \frac{501}{100}$$

$$3.27 = 3\frac{27}{100} = \frac{327}{100}$$

Dividing Integers

Now we can divide any integer by any other integer except 0. All we have to do is write a fraction with the dividend (the number we are dividing *into*) as the numerator (top) and the divisor (the number we are dividing *by*) as the denominator (bottom).

$$10 \div 3 = \frac{10}{3}$$

Do each division problem. If the divisor goes evenly into the dividend, write your answer as an integer. Otherwise, write it as a fraction.

$$\begin{array}{llll} 12 \div -2 = -6 & 15 \div 3 = \frac{15}{3} = 5 & 15 \div 4 = \frac{15}{4} & -8 \div -3 = \frac{-8}{-3} = \frac{8}{3} \\ 40 \div 7 = \frac{40}{7} & -7 \div 2 = \frac{-7}{2} & 1 \div -6 = \frac{1}{-6} & 54 \div 7 = \frac{54}{7} \\ -3 \div 5 = \frac{-3}{5} & -45 \div -9 = \frac{-45}{-9} = 5 & -7 \div 6 = \frac{-7}{6} & -2 \div 19 = \frac{-2}{19} \end{array}$$

A fraction can be positive or negative. To find the sign, just follow the rules for division. When division is written using a fraction bar, the rules look like this:

$$\frac{\text{POSITIVE}}{\text{POSITIVE}} = \text{POSITIVE}$$

$$\frac{\text{NEGATIVE}}{\text{POSITIVE}} = \text{NEGATIVE}$$

$$\frac{\text{POSITIVE}}{\text{NEGATIVE}} = \text{NEGATIVE}$$

$$\frac{\text{NEGATIVE}}{\text{NEGATIVE}} = \text{POSITIVE}$$

If a fraction is positive, we will write it with no signs. If a fraction is negative, we will write it with the negative sign on top.

$$\frac{-3}{4} \text{ is positive, so we will write } \frac{3}{4}.$$

$$\frac{3}{-4} \text{ is negative, so we will write } \frac{-3}{4}.$$

Do each division problem. Write your answer as a positive or negative fraction.

$$\begin{array}{llll} 13 \div -3 = \frac{-13}{3} & -4 \div -7 = \frac{-4}{-7} = \frac{4}{7} & 15 \div -2 = \frac{15}{-2} & 18 \div -5 = \frac{18}{-5} \\ -3 \div 10 = \frac{-3}{10} & 1 \div -5 = \frac{1}{-5} & -1 \div 9 = \frac{-1}{9} & 40 \div 29 = \frac{40}{29} \\ -9 \div -10 = \frac{-9}{-10} = \frac{9}{10} & 9 \div -5 = \frac{9}{-5} & 12 \div -7 = \frac{12}{-7} & -3 \div -100 = \frac{-3}{-100} = \frac{3}{100} \\ 12 \div 11 = \frac{12}{11} & 4 \div 5 = \frac{4}{5} & -9 \div -14 = \frac{-9}{-14} = \frac{9}{14} & -20 \div 7 = \frac{-20}{7} \end{array}$$

Divide. Write your answer as an integer or as a positive or negative mixed number.

4 goes into 9 2 times with a remainder of 1.

7 goes into 12 1 time with a remainder of 5. The answer is negative.

$$9 \div 4 = 2\frac{1}{4}$$

$$-12 \div 7 = -1\frac{5}{7}$$

$$-18 \div -7 = 2\frac{4}{7}$$

$$50 \div -5 = -10$$

$$30 \div 9 = 3\frac{3}{9} = 3\frac{1}{3}$$

$$-7 \div -6 = 1\frac{1}{6}$$

$$28 \div -4 = -7$$

$$-18 \div 5 = -3\frac{3}{5}$$

$$-60 \div 4 = -15$$

$$-80 \div -10 = 8$$

$$46 \div -2 = -23$$

$$64 \div -3 = -21\frac{1}{3}$$

$$25 \div -7 = -3\frac{4}{7}$$

$$-63 \div -9 = 7$$

$$-100 \div 3 = -33\frac{1}{3}$$

$$0 \div -6 = 0$$

$$20 \div 3 = 6\frac{2}{3}$$

$$-14 \div -5 = 2\frac{4}{5}$$

$$-11 \div 4 = -2\frac{3}{4}$$

$$-25 \div -4 = 6\frac{1}{4}$$

$$-37 \div 10 = -3\frac{7}{10}$$

Divide. This time write your answer as a positive or negative decimal.

$$-3 \div 10 = \frac{-3}{10} = -0.3$$

$$107 \div 100 = \frac{107}{100} = 1\frac{7}{100} = 1.07$$

$$-8 \div -10 = \frac{8}{10} = .8$$

$$-41 \div 100 = \frac{-41}{100} = -.41$$

$$39 \div 10 = \frac{39}{10} = 3.9$$

$$-253 \div -100 = \frac{253}{100} = 2.53$$

3.75
4/15.00

$$15 \div 4 = 3.75$$

$$-26 \div 5 = -\frac{26}{5} = -5.2$$

$$-17 \div 2 = -\frac{17}{2} = -8.5$$

$$30 \div -4 = -\frac{30}{4} = -7.5$$

$$-100 \div -8 = \frac{100}{8} = 12.5$$

$$-12 \div 5 = -\frac{12}{5} = -2.4$$

Equations with Rational Solutions

Now we can use the Division Principle to solve equations even when the answer is not an integer. Solve each equation. Write your answer as a fraction or as a mixed number.

$\frac{7x}{7} = \frac{-25}{7}$ $x = -3\frac{4}{7}$	$\frac{9x}{9} = \frac{40}{9}$ $x = 4\frac{5}{9}$	$\frac{-4x}{-4} = \frac{17}{-4}$ $x = -4\frac{1}{4}$
$2x - 5 = 14$ $\begin{array}{r} +5 \quad +5 \\ 2x - 5 = 14 \\ \hline 2x = 19 \\ \hline x = 9\frac{1}{2} \end{array}$	$3x + 7 = -4$ $\begin{array}{r} -7 \quad -7 \\ 3x + 7 = -4 \\ \hline 3x = -11 \\ \hline x = -9\frac{2}{3} \end{array}$	$-5x + 1 = 15$ $\begin{array}{r} -1 \quad -1 \\ -5x + 1 = 15 \\ \hline -5x = 14 \\ \hline x = -2\frac{4}{5} \end{array}$
$x - 3 = 8x + 5$ $\begin{array}{r} -x - 5 \quad -x - 5 \\ x - 3 = 8x + 5 \\ \hline -8 = 7x \\ \hline x = -\frac{8}{7} \end{array}$	$-2(x - 5) = 7$ $\begin{array}{r} -2x + 10 = 7 \\ -10 \quad -10 \\ -2x + 10 = 7 \\ \hline -2x = -3 \\ \hline x = \frac{3}{2} \end{array}$	$x - 5x + 7 = -8$ $\begin{array}{r} -7 \quad -7 \\ x - 5x + 7 = -8 \\ \hline -4x = -15 \\ \hline x = 3\frac{3}{4} \end{array}$

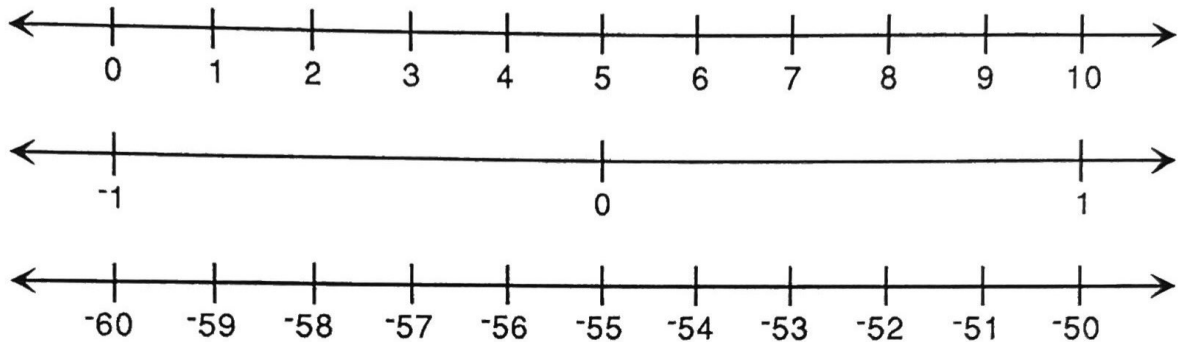
Solve these equations, too. This time if your answer is not an integer, write it as a decimal.

$\frac{4x}{4} = \frac{-10}{4}$ $x = -2.5$ <p style="text-align: center;">(2.5) 4)10.0</p>	$\frac{-5x}{-5} = \frac{18}{-5}$ $x = -3.6$	$5 \cdot \frac{-2x}{5} = 7.5$ $\begin{array}{r} -2x = 3.5 \\ -2 \quad -2 \\ -2x = 3.5 \\ \hline x = -1.75 \end{array}$
$7x - 7 = 3x + 20$ $\begin{array}{r} -3x + 7 \quad -3x + 7 \\ 7x - 7 = 3x + 20 \\ \hline 4x = 27 \\ \hline x = 6.75 \end{array}$	$x - 9 = 6x + 7$ $\begin{array}{r} -x - 7 \quad -x - 7 \\ x - 9 = 6x + 7 \\ \hline -16 = 5x \\ \hline x = -3.2 \end{array}$	$5 \cdot \frac{10x - 2}{5} = 3.5$ $\begin{array}{r} 10x - 2 = 15 \\ +2 \quad +2 \\ 10x = 17 \\ \hline x = 1.7 \end{array}$
$3(x - 5) = x - 20$ $\begin{array}{r} 3x - 15 = x - 20 \\ -x + 15 \quad -x + 15 \\ 3x - 15 = x - 20 \\ \hline 2x = -5 \\ \hline x = -2.5 \end{array}$	$4(x + 6) = 23$ $\begin{array}{r} 4x + 24 = 23 \\ -24 \quad -24 \\ 4x + 24 = 23 \\ \hline 4x = -1 \\ \hline x = -.25 \end{array}$	$2(x - 3) + x = 9$ $\begin{array}{r} 2x - 6 + x = 9 \\ +6 \quad +6 \\ 3x = 15 \\ \hline x = 5 \end{array}$

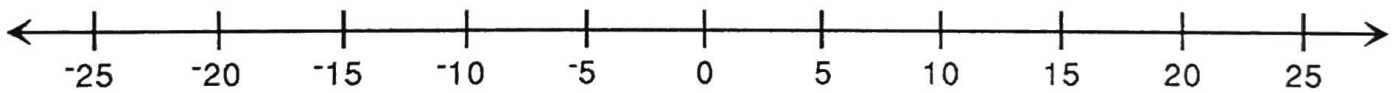
Number Lines

In Book 1 we used **number lines** to help us think about adding and multiplying integers. The football field was a kind of number line. Rulers and the scales on thermometers are also number lines.

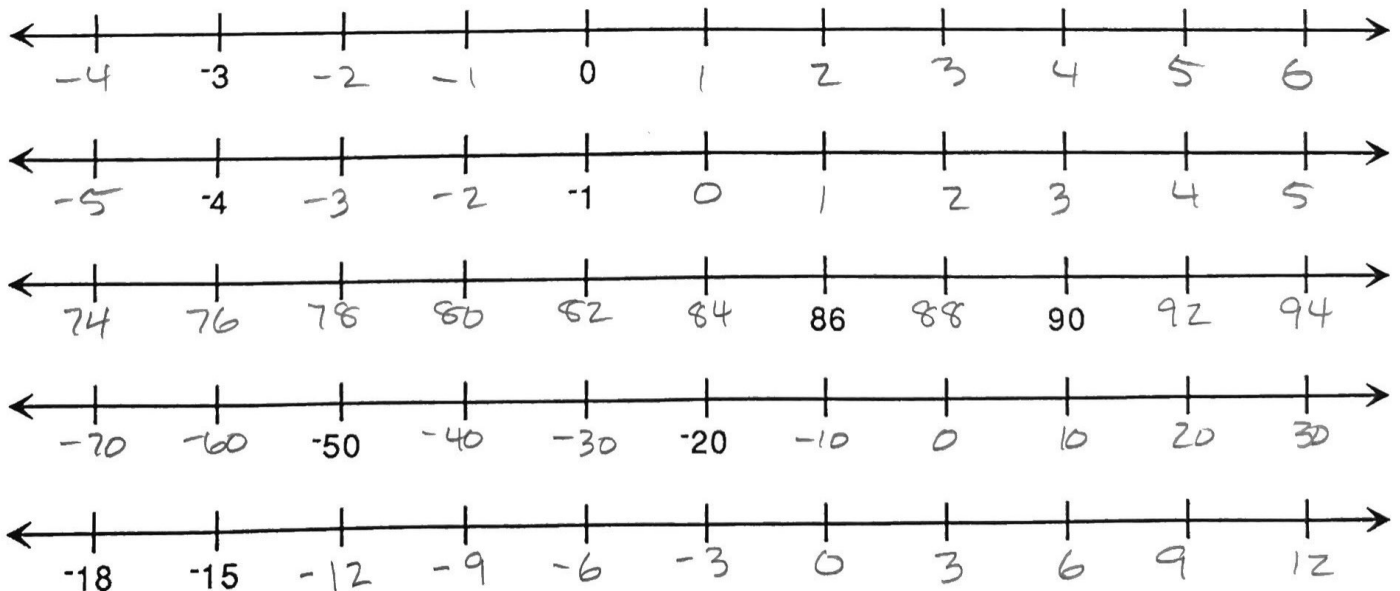
To make a number line we draw a line and divide it into sections of equal length called **units**. Then we number the points which separate the units. Here are three number lines:



The arrows on the ends of each number line show that the number line keeps going. We can start with any number as long as we number the points in order (usually from left to right). Sometimes we do not show every unit. This number line only shows every fifth unit:



Here are some number lines for you to finish numbering:



Make a number line showing all the integers from -5 to 5.

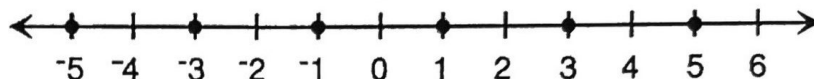
Graphing Integers

We can use a number line to picture a set of numbers. On the number line we make a dot to show each number in the set. This is called a **graph** of the set. A graph can help you see a pattern or answer a question. If a pattern continues forever to the left or right, we fill in the arrow that points in that direction.

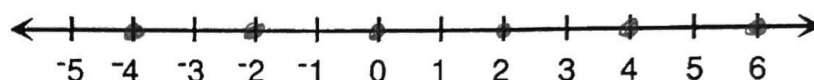
Graph each set of integers below.

This pattern suggests that 0 is even.

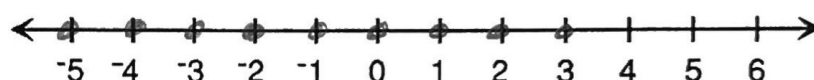
Odd integers:



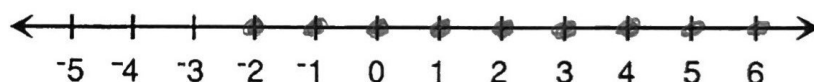
Even integers:



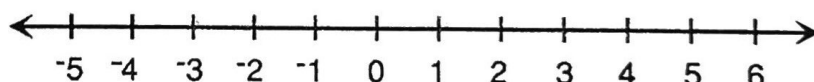
Integers less than 4:



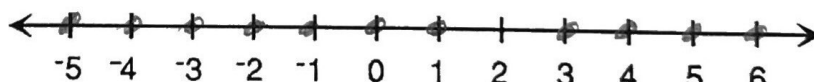
Integers greater than -3:



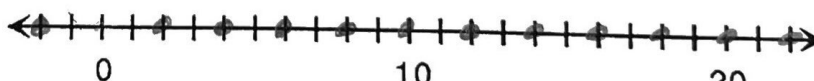
Integers between -3 and -4:



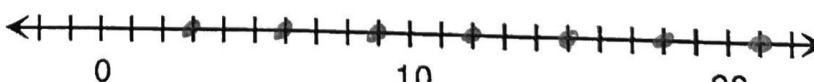
Integers not equal to 2:



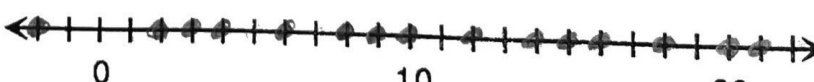
Integers divisible by 2:



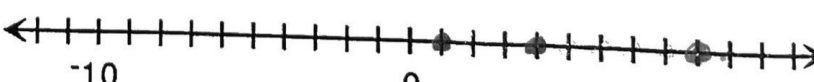
Integers divisible by 3:



Integers divisible by 2 and 3:



The squares of integers:



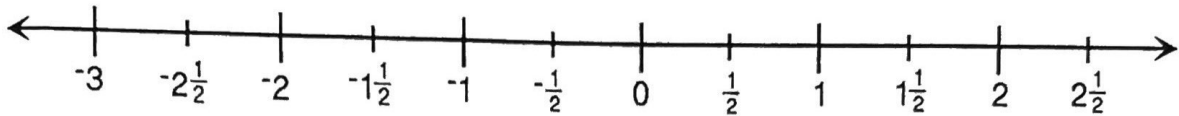
Integers with squares which are less than 10:



Did you notice any interesting patterns in the graphs you made?

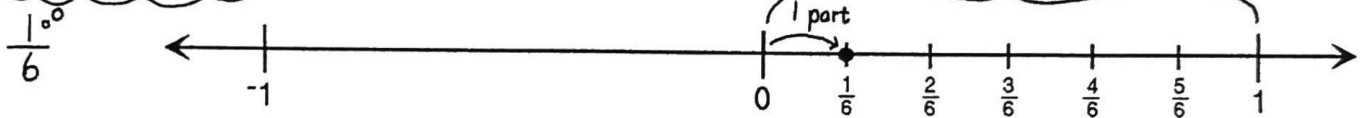
Graphing Rational Numbers

Integers are not the only points on a number line. On the number line below we have also labeled the points halfway between each integer and the next.

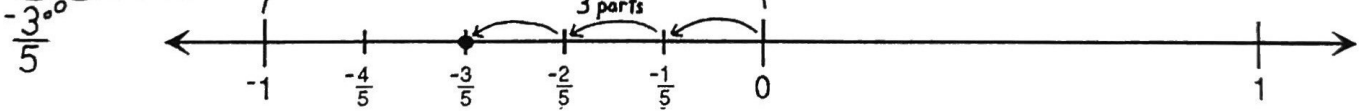


In fact, there is a point on the number line for *each* rational number. To find this point, first write the rational number as a fraction. The denominator of the fraction tells how many parts to divide each unit of the number line into. The numerator tells how many parts to count off to the right of 0 (if the number is positive) or to the left of 0 (if the number is negative) to find the point. Here's how to find $\frac{1}{6}$, $\frac{3}{5}$, and $\frac{19}{7}$.

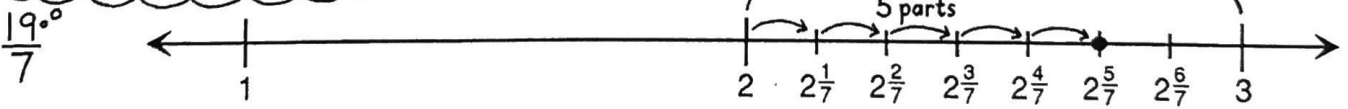
$\frac{1}{6}$ is between 0 and 1.



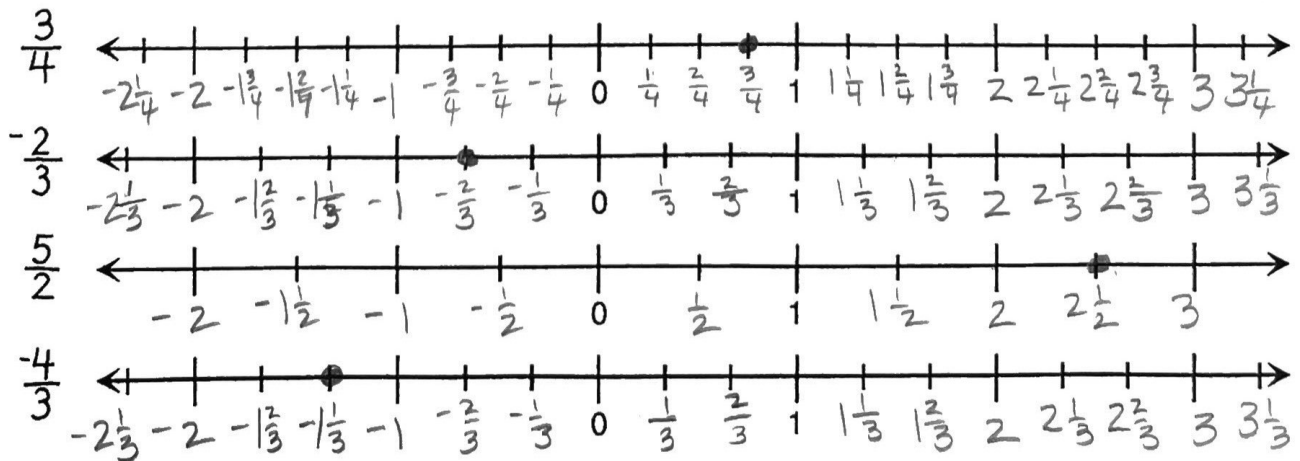
$\frac{3}{5}$ is between 0 and 1.



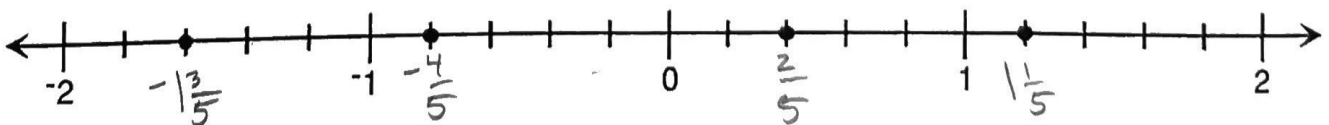
$\frac{19}{7} = 2\frac{5}{7}$, so it is between 2 and 3.



On each number line, first finish labeling the points. Then graph the rational number at the left.

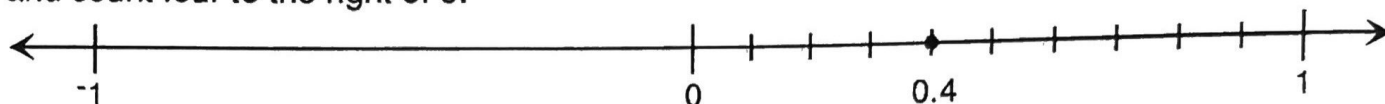


Label each rational number shown on the number line below.

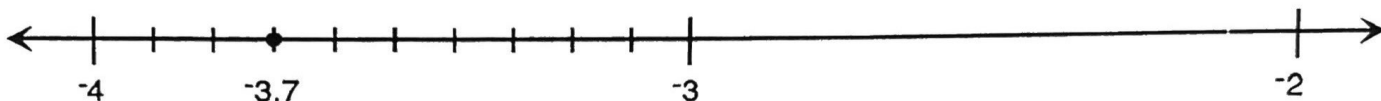


Each decimal is a rational number (unless it goes on forever without repeating), so it also has a place on the number line. To find the point for a decimal, think of it as a fraction or mixed number.

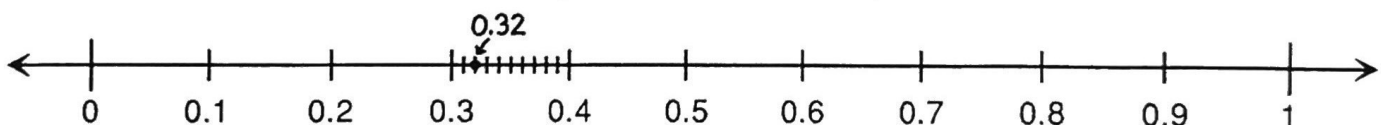
0.4 is the same as $\frac{4}{10}$. This number is between 0 and 1 so we divide that unit into ten parts and count four to the right of 0.



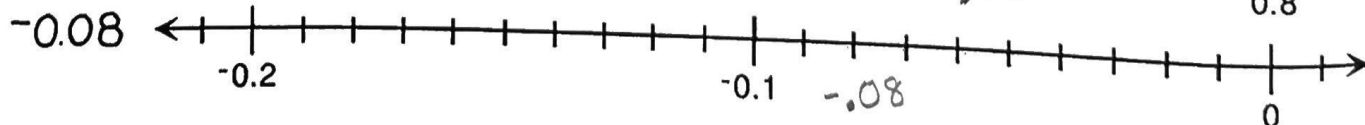
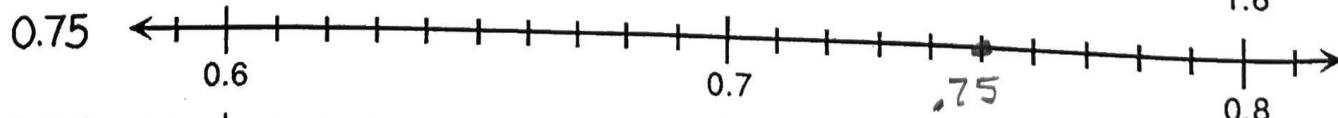
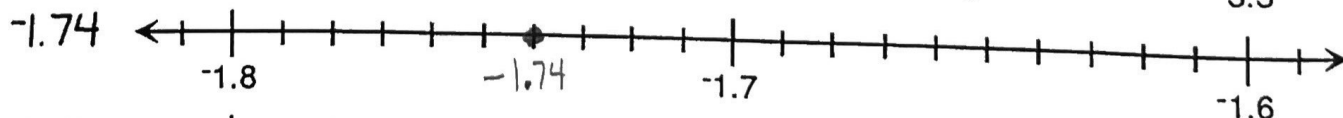
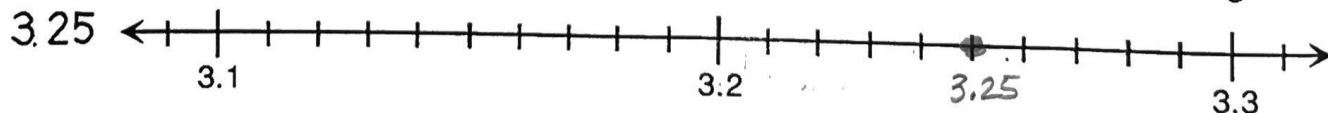
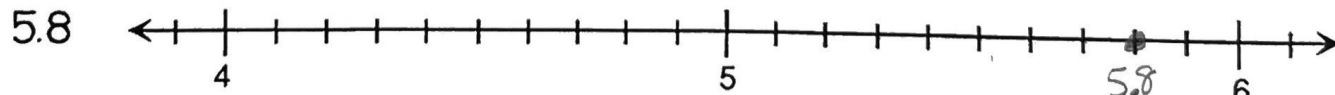
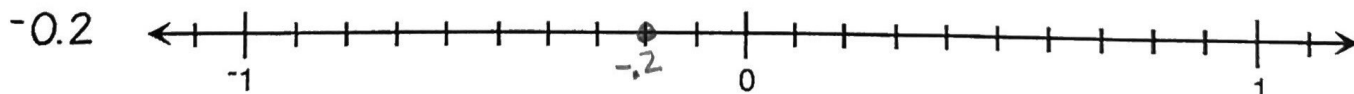
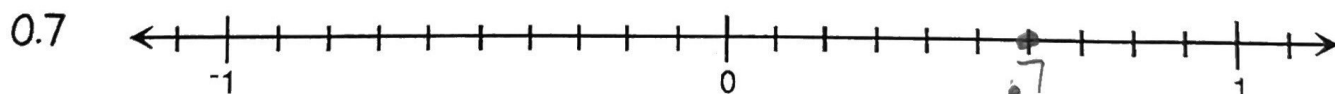
-3.7 is equal to $-3\frac{7}{10}$. This number is between -3 and -4 so we divide that unit into ten parts and count seven units to the left of -3.



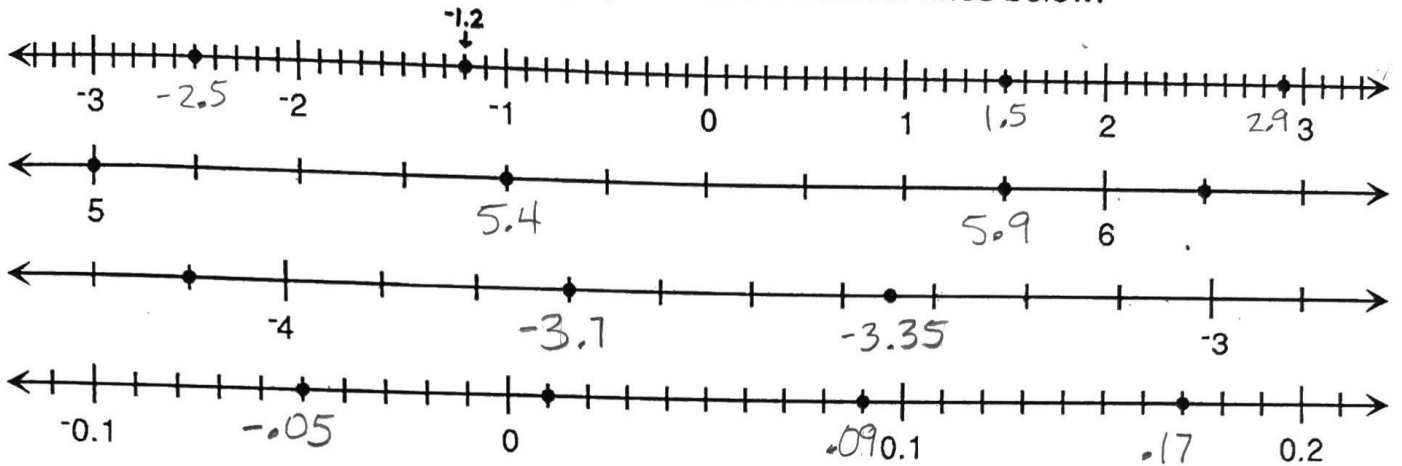
For hundredths we could divide the unit into a hundred parts, but to save time it makes sense to divide it into tenths first and then to divide only one of the tenths into ten parts. To graph 0.32 this way we first notice that it is between 0.3 and 0.4. Then we divide the section between 0.3 and 0.4 into ten parts. Each of these parts is a hundredth of the unit.



Graph each decimal below.

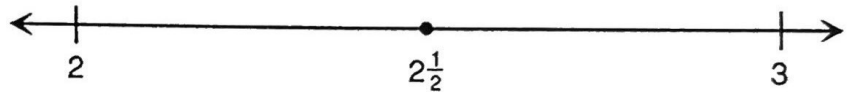


Find a decimal name for each point graphed on the number lines below.

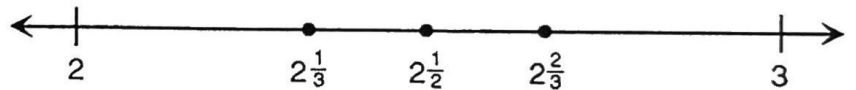


Imagine making a graph of all the rational numbers between 2 and 3.

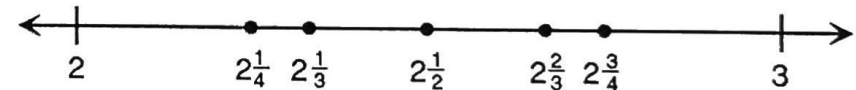
First we would graph the halves,



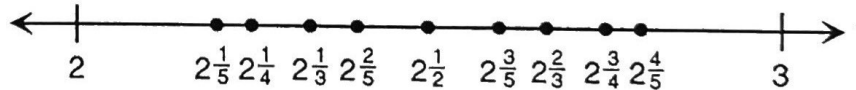
then the thirds,



then the fourths,



then the fifths,



and so on . . .

We would never be finished! Soon the line would be so crowded with dots that you couldn't tell one from another. So when we want to show *all* the rational numbers between 2 and 3 we just shade the whole section of the line between those numbers.



Whenever we say "between" we will mean "not including the endpoints."

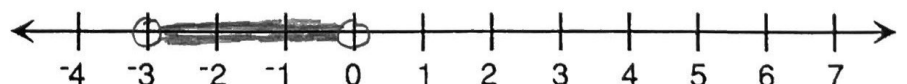
We have used hollow dots at 2 and 3 to show that those numbers are not included.

You graph all the rational numbers which are:

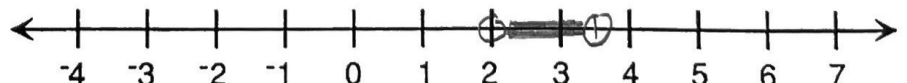
between -1 and 4



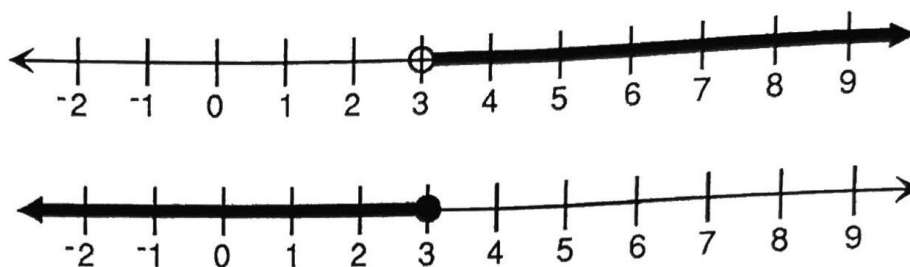
between -3 and 0



between 2 and 3.5



Can you tell what sets have been graphed?



The first graph shows all rational numbers which are greater than 3. The hollow dot shows that 3 is not included.

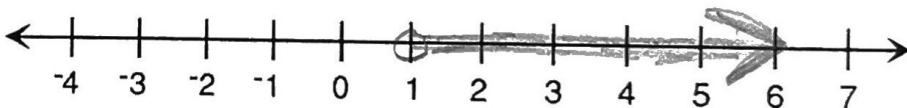
The second graph shows all rational numbers which are less than or equal to 3. This time 3 is included, so we have used a solid dot. On both graphs the arrows have been filled in to show that the graphs continue.

Graph all the rational numbers which are:

less than 6



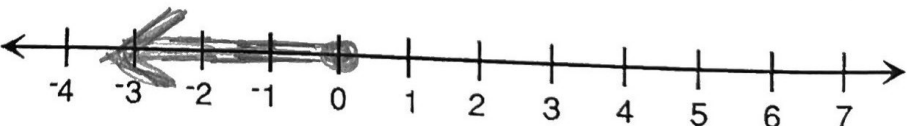
greater than 1



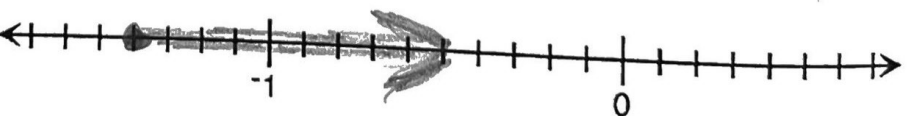
greater than or equal to 4



less than or equal to 0



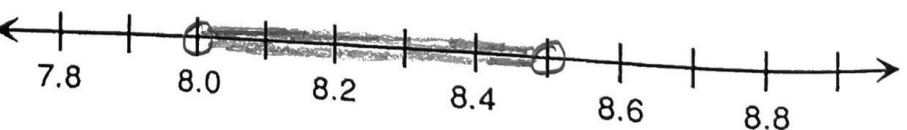
greater than or equal to -1.4



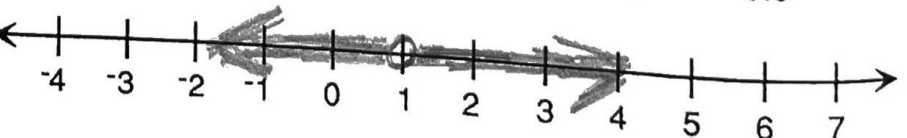
less than or equal to 0.5



between 8 and 8.5



not equal to 1



Inequalities

In Book 3 we worked with equations. Remember that an equation is a sentence about numbers being *equal*, like $x + -4 = 5$.

Another kind of sentence is an **inequality** — a sentence about numbers being *unequal*. Here are two examples of inequalities:

$x + -4 < 5$ means " $x + -4$ is less than 5."

$x + -4 > 5$ means " $x + -4$ is greater than 5."

Sometimes we combine two symbols to make a new symbol. The symbol \leq means "is less than or equal to." And the symbol \geq means "is greater than or equal to." Sentences using these combined symbols are also inequalities.

$x + -4 \leq 5$ means " $x + -4$ is less than or equal to 5."

$x + -4 \geq 5$ means " $x + -4$ is greater than or equal to 5."

To get used to using $<$, $>$, \leq and \geq , read the following statements carefully.

Each of these is true:

$$10 > 6 \quad 5 \geq 5 \quad -8 < 8 \quad 12 = 12 \quad -4 \geq -7$$

$$9 \geq -2 \quad 5 \leq 5 \quad 8 > -8 \quad 12 \geq 12 \quad -7 \leq -4$$

Each of these is false:

$$3 > 4 \quad -6 < -6 \quad -9 \geq 9 \quad 1 \leq 0 \quad 1 \geq 2$$

$$3 \geq 4 \quad -6 > -6 \quad 9 \leq -9 \quad 0 < 0 \quad 2 > 2$$

Mark each statement below true (T) or false (F).

$$18 > 2 \quad \text{T}$$

$$-14 \leq 1 \quad \text{T}$$

$$7 < 7 \quad \text{F}$$

$$18 \geq 2 \quad \text{T}$$

$$-14 \leq -15 \quad \text{F}$$

$$7 \leq 7 \quad \text{T}$$

$$18 \geq 18 \quad \text{T}$$

$$-14 \leq -14 \quad \text{T}$$

$$-7 \leq 7 \quad \text{T}$$

$$-3 \leq -3 \quad \text{T}$$

$$0 > -8 \quad \text{T}$$

$$10 \leq 24 \quad \text{T}$$

$$-3 < -3 \quad \text{F}$$

$$0 \geq -8 \quad \text{T}$$

$$13 > -2 \quad \text{T}$$

$$-3 < 0 \quad \text{T}$$


$$-8 \leq 0 \quad \text{T}$$

$$-4 \geq -1 \quad \text{F}$$


 is the international road sign for "Passing Permitted."

 is the sign for "No Passing."


Can you figure out what each sign below means?

 is the sign for


No Smoking

 is the sign for

No camping

 is the sign for

No picnic tables (eating)

 is the sign for

No bike riding

A slash is used on international road signs to mean "No." We use the same idea to make up new symbols in algebra. In these symbols the slash means "is not."

$7 \neq 4$ means "7 is not equal to 4."

$0 \not> 5$ means "0 is not greater than 5."

$2 \not< 2$ means "2 is not less than 2."

$10 \not\geq 12$ means "10 is not greater than or equal to 12."

$3 \not\leq 0$ means "3 is not less than or equal to 0."

You write the meaning of each sentence below.

$8 \not< 4$ means

8 is not less than 4

$0 \not> 3$ means

0 is not greater than 3

$5 \not\geq 6$ means

5 is not greater than or equal to 6

$-10 \neq 10$ means

-10 is not equal to 10

$x + 4 \neq 9$ means

$x + 4$ is not equal to 9

$-2x \not< 10$ means

$-2x$ is not less than 10

$\frac{x}{3} \not\geq 5$ means

$\frac{x}{3}$ is not greater than or equal to 5

A number is a **solution** of an inequality if it makes the inequality true when you try it in place of x .

These numbers are solutions of $x + -4 < 5$:

7 because $7 + -4 < 5$

-1 because $-1 + -4 < 5$

0 because $0 + -4 < 5$

These numbers are *not* solutions of $x + -4 < 5$:

10 because $10 + -4 \not< 5$

9 because $9 + -4 \not< 5$

25 because $25 + -4 \not< 5$

Try to find at least five integers which are solutions for each equation or inequality. If there aren't five integer solutions, list as many as you can find.

$x > -3$ 0, -2, 3, 5, 10

$x + 8 = 10$ 2

$x \neq 0$ 3, 1, 7, 9, 12

$x + 8 < 10$ 1, 0, -6, -4, -1

$x \neq 1$ 2, 5, -6, 12, 19

$x + 8 > 10$ 3, 5, 7, 12, 73

$x \leq x$ 5, 0, -6, 7, 25

$x + 8 \geq 10$ 2, 3, 9, 8, 16

$x < x$ No solution

$x + 8 \leq 10$ 2, 1, 0, -1, -12

$x \neq x$ No solution

$x + 8 \neq 10$ 1, 0, 12, -4, -6

$x = x$ -9, 0, 7, 10, 152

$x + 8 \neq 10$ 1, -2, 0, -6, -5

$x > x$ No solution

$x + 8 \neq 10$ 3, 5, 12, 16, 21

$x < x^2$ 3, 5, 12, 8, 92

$x + 8 \neq 10$ 4, 9, 17, 25, 63

Absolute Value

When we multiply or divide integers we get the *amount* of the answer by multiplying or dividing and the *sign* of the answer by following the rules for signs. The amount of a number is often called its **absolute value**. We put the symbol $| |$ around a number when we want to talk about its absolute value.

$$|6| = 6 \quad \text{means} \quad \text{"The absolute value of 6 is 6."}$$

$$|-6| = 6 \quad \text{means} \quad \text{"The absolute value of -6 is 6."}$$

$$|0| = 0 \quad \text{means} \quad \text{"The absolute value of 0 is 0."}$$

Finding the absolute value of a number is easy. Just get rid of its sign. You find each absolute value below.

$$|-4| = 4 \quad |-19| = 19 \quad |-\frac{2}{3}| = \frac{2}{3} \quad |3-9| = |-6| = 6$$

$$|7| = 7 \quad |19| = 19 \quad |-1.3| = 1.3 \quad |4(-5)| = |-20| = 20$$

$$|-10| = 10 \quad |0.19| = .19 \quad |0.027| = .027 \quad |(-6)(-4)| = |24| = 24$$

Use trial and error to find as many solutions as you can for each equation below.

Both 5 and -5 have absolute values of 5.

$ x = 5$	<u>5, -5</u>	$ x+2 = 4$	<u>2, -6</u>
$ x = 8$	<u>8, -8</u>	$ 3x = 27$	<u>9, -9</u>
$ x = 0$	<u>0</u>	$ x-5 = 2$	<u>3, 7</u>
$ x = -5$	<u>No solution</u>	$ x-8 = 0$	<u>8</u>

For each inequality below, try to find at least five integers which are solutions.

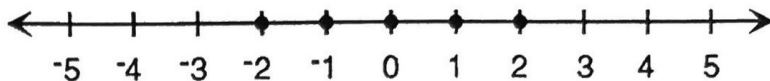
7, 10 and 12 have absolute values greater than 6, but so do -7, -10 and -12.

$ x > 6$	<u>7, 10, 12, -7, -10, -12</u>	$ x \neq 7$	<u>-9, 9, -12, 12, 16</u>
$ x < 6$	<u>-3, 3, -5, 5, 1</u>	$ x \neq 10$	<u>-2, 2, 0, 7, 5</u>
$ x \geq 0$	<u>0, 1, 2, 3, 4</u>	$ x > x$	<u>No solution</u>
$ x \neq 3$	<u>-2, 5, 9, 8, -5</u>	$ x \leq 2$	<u>-2, 2, 1, 0, -1</u>

Graphing Inequalities

Look at the last equation on the previous page. The five integers which are solutions are -2, -1, 0, 1 and 2. We could easily make a graph of this set of solutions.

$$|x| \leq 2$$

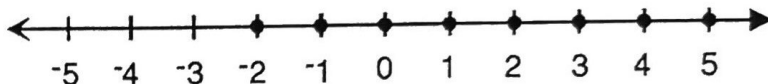


Some inequalities, like $x > -3$, have an infinite number of solutions. It would be impossible to list all the integers which are solutions, but we could show the solution set by *starting* a list and then using three dots to show that it continues on and on.

$$x > -3 \quad \{-2, -1, 0, 1, 2, 3, \dots\}$$

We could also graph the set of solutions using a darkened arrow on the right to show that the dots continue to the right.

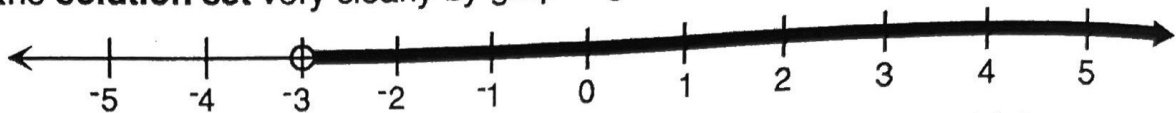
$$x > -3 \quad \{-2, -1, 0, 1, 2, 3, \dots\}$$



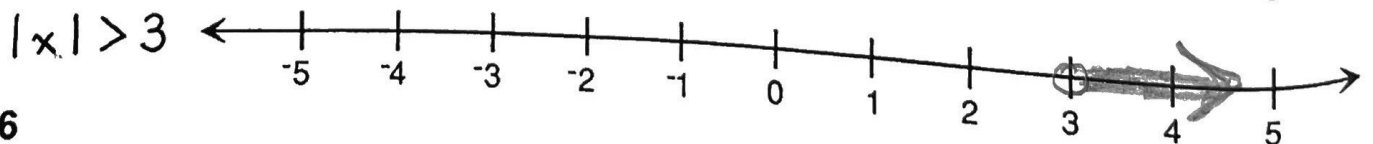
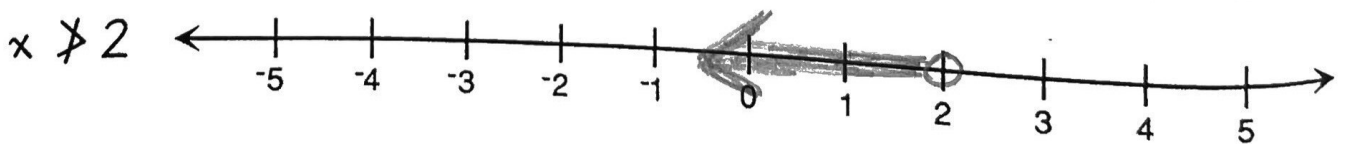
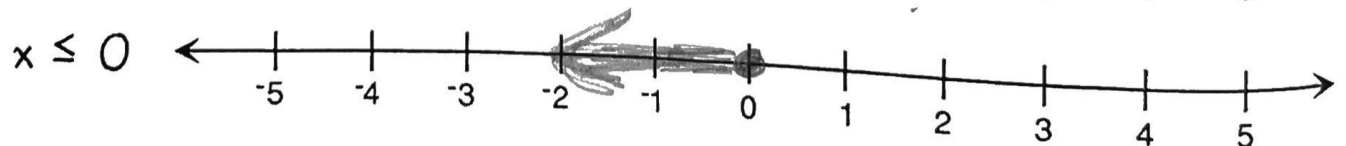
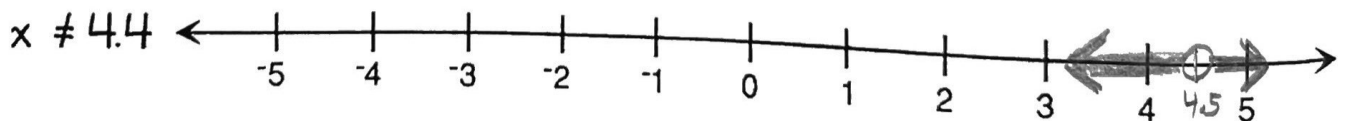
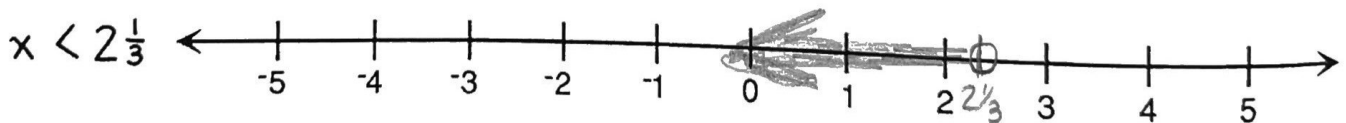
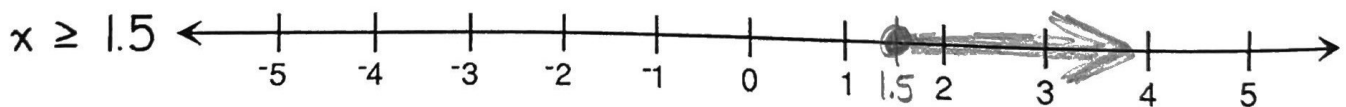
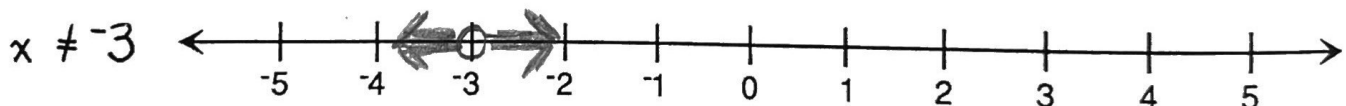
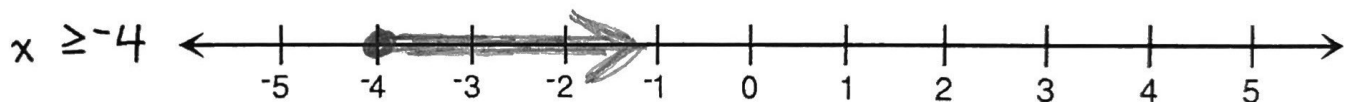
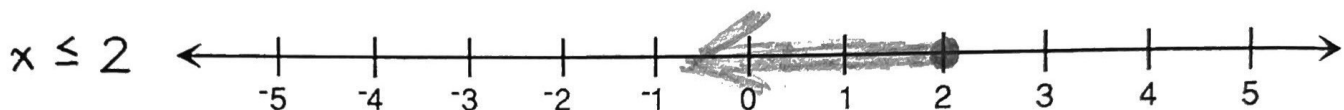
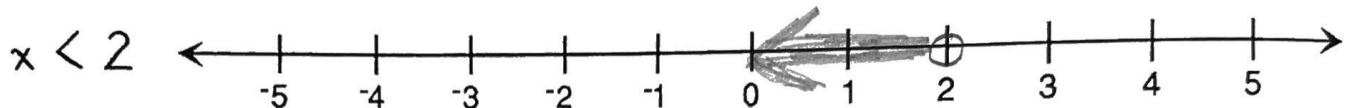
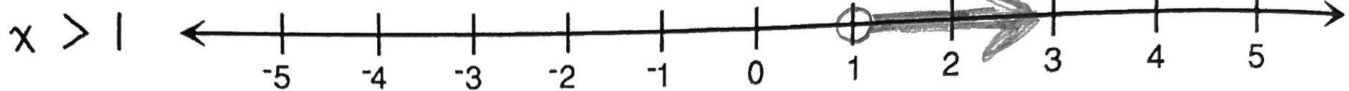
For each inequality, show the integers which are solutions in two ways: by making a list and by graphing.

	List	Graph
$x < 1$	$\{0, -1, -2, -3, \dots\}$	
$x > 5$	$\{6, 7, 8, 9, \dots\}$	
$x \leq -3$	$\{-3, -4, -5, \dots\}$	
$x \geq -4$	$\{-4, -3, -2, -1, \dots\}$	
$x > 210$	$\{211, 212, 213, \dots\}$	
$x \leq 0$	$\{0, -1, -2, -3, \dots\}$	
$ x < 4$	$\{-3, 3, -2, 2, \dots\}$	
$ x \geq 2$	$\{2, -2, 3, -3, 4, -4, \dots\}$	
$x + 1 \geq 5$	$\{4, 5, 6, 7, \dots\}$	

It would be impossible to list all the rational numbers which are solutions of the inequality $x > -3$. -3 is not a solution, but every rational number greater than -3 is. We can show the **solution set** very clearly by graphing.



For each equation or inequality below, graph the set of all rational numbers which are solutions.

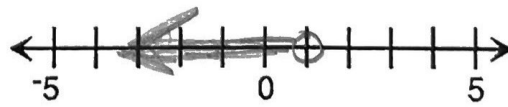
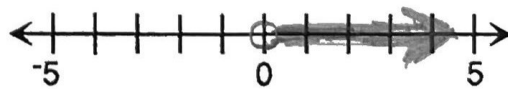
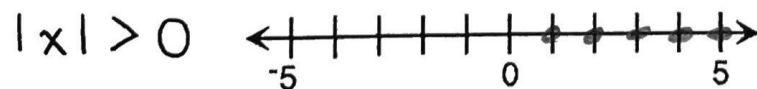
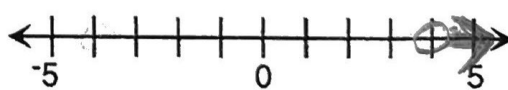
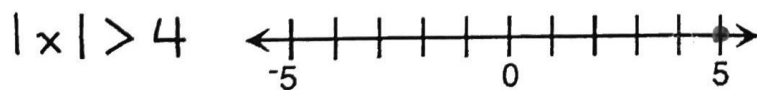
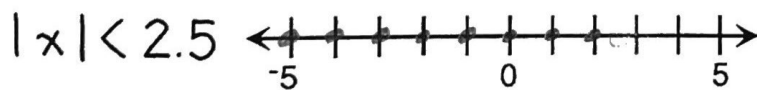
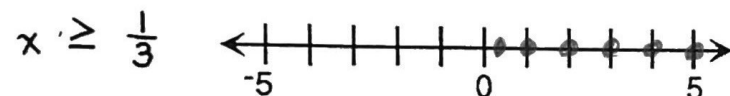
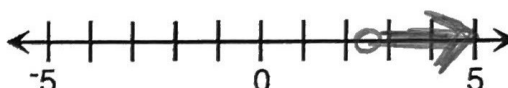
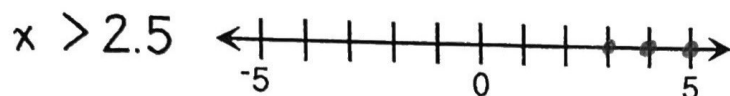
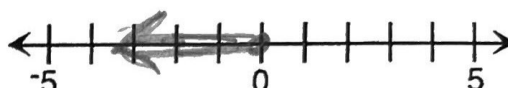
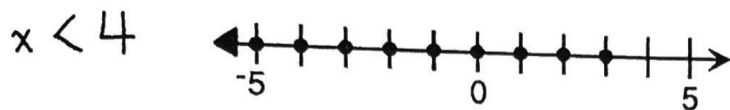


The graph of an inequality depends on what kinds of numbers we allow as solutions. The set of numbers we allow as solutions is called the **replacement set**.

In the first problem below we have shown what the graph of $x < 4$ looks like when only integers are allowed as solutions and what it looks like when all rational numbers are allowed as solutions. Make a graph of each inequality for each replacement set.

Integers

Rational Numbers



From now on in this book we'll always use rational numbers as our replacement set.

Solving Inequalities

We can tell if a number is a solution of an equation or inequality by substituting that number for the variable and seeing whether the result is true or false.

Is 8 a solution of $2x + 5 < x - 15$? $2x + 5 < x - 15$

We can substitute 8 for x to find out:

$$2 \cdot 8 + 5 < 8 - 15$$

$$16 + 5 < 8 - 15$$

$$21 < -7$$

So 8 is not a solution. false

Luckily we do not have to substitute every time we want to check a possible solution. We can use the Addition Principle to help us solve an inequality just as we used it to solve equations. The Addition Principle can help us find a simpler inequality with the same solution set. This is called **solving** the inequality. Look at this example:

$$2x + 5 < x - 15$$

$$x + 5 < -15$$

$$x < -20$$

Only numbers less than -20 are solutions, so 8 could not be a solution. Neither could any other positive number.

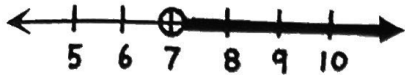
Solve each inequality below using the Addition Principle.

$x + 19 \geq 3$ $x \geq -16$	$x - 4 < 12$ $+4 \quad +4$ $x < 16$	$x + 9 \leq -6$ $-9 \quad -9$ $x \leq -15$
$-2 + x < 6$ $+2 \quad +2$ $x < 8$	$-11 + x \geq 1$ $-11 \quad -11$ $x \geq -10$	$x - 5 < -3$ $+5 \quad +5$ $x < 2$
$5 > x + 4$ $1 > x$ $x < 1$ <p style="border: 1px solid black; border-radius: 50%; padding: 5px; display: inline-block;">If 1 is bigger than x, then x is less than 1.</p>	$10 < x + 6$ $-6 \quad -6$ $4 < x$ or $x > 4$	$12 \geq x - 7$ $+7 \quad +7$ $19 \geq x$ $x \leq 19$
$5x + 1 > 4x + 15$ $x + 1 > 15$ $x > 14$	$7x - 9 < 6x + 11$ $-6x + 9 \quad -6x + 9$ $x < 20$	$x - 5 \leq 2x + 8$ $-x - 8 \quad -x - 8$ $-13 \leq x$ or $x \geq -13$

Solve each inequality below using the Addition Principle. Draw a graph of each solution set.

$$x - 7 > 0$$

$$x > 7$$



$$x - 9 < -9$$

$$+9 \quad +9$$

$$x < 0$$



$$6 \leq x + 12$$

$$-12 \quad -12$$

$$-6 \leq x \text{ or } x \geq -6$$



$$2x + 1 > x - 4$$

$$-x \quad -x$$

$$x > -5$$



$$x + 16 < 2x + 20$$

$$-x \quad -x$$

$$-4 < x \text{ or } x > -4$$



$$5x - 6 \leq 4x + 1$$

$$-4x \quad -4x$$

$$x \leq 7$$



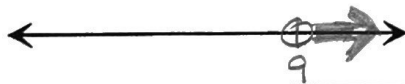
$$3(x - 3) > 2x$$

$$3x - 9 > 2x$$

$$-3x \quad -3x$$

$$\frac{-9}{-1} > \frac{-x}{-1}$$

$$9 < x \text{ or } x > 9$$



$$x - 4 + 11 \leq 26$$

$$x + 7 \leq 26$$

$$-7 \quad -7$$

$$x \leq 19$$



$$2(x - 1) < 3(x + 4)$$

$$2x - 2 < 3x + 12$$

$$-2x \quad -2x$$

$$-14 < x \text{ or } x > -14$$

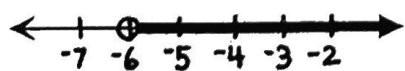


$$x^2 - 5 < x^2 + x + 1$$

$$-5 < x + 1$$

$$-6 < x$$

$$x > -6$$



$$\frac{x^2}{-x^2} + x + 8 \geq \frac{x^2}{-x^2} - 4$$

$$-8 \quad -8$$

$$x \geq -12$$



$$2x^2 + 22 > 2x^2 + x - 9$$

$$2x^2 + 9 \quad -2x^2 + 9$$

$$31 > x \text{ or } x < 31$$



$$(x + 2)(x - 2) < x^2 + x$$

$$x^2 - 4 < x^2 + x$$

$$-4 < x$$

$$x > -4$$



$$(x + 2)(x - 1) \geq x^2$$

$$x^2 + x - 2 \geq x^2$$

$$-x^2 \quad -x^2$$

$$x - 2 \geq 0$$

$$+2 \quad +2$$

$$x \geq 2$$



$$(x - 4)(x + 5) < x^2 - 19$$

$$x^2 - 4x + 5x - 20 < x^2 - 19$$

$$-x^2 \quad -x^2$$

$$x - 20 < -19$$

$$+20 \quad +20$$

$$x < 1$$



Maybe you are wondering whether there are Multiplication and Division Principles for Inequalities. The answer is yes, but they aren't quite the same as the Multiplication and Division Principles for Equations. When we multiply or divide both sides of an inequality by a *positive* number, we do get an equivalent inequality. But when we multiply or divide both sides by a *negative* number, we must *reverse the inequality sign* to get an equivalent inequality. Look at these sentences to see why:

Multiplying by 2:	$-6 < 10 \leftarrow \text{true}$ $-6 \cdot 2 < 10 \cdot 2 \leftarrow \text{true}$ $-12 < 20 \swarrow$	Dividing by 2:	$-6 < 10 \leftarrow \text{true}$ $-\frac{6}{2} < \frac{10}{2} \leftarrow \text{true}$ $-3 < 5 \swarrow$
Multiplying by -2:	$-6 < 10 \leftarrow \text{true}$ $-6 \cdot (-2) < 10 \cdot (-2) \leftarrow \text{false}$ $12 < -20 \swarrow$ $12 > -20 \leftarrow \text{true with } < \text{ switched to } >$	Dividing by -2:	$-6 < 10 \leftarrow \text{true}$ $-\frac{-6}{-2} < \frac{-10}{-2} \leftarrow \text{false}$ $3 < -5 \swarrow$ $3 > -5 \leftarrow \text{true with } < \text{ switched to } >$

Solve each inequality using the Multiplication and Division Principles. Remember to switch the direction of the inequality sign if you multiply or divide by a negative number.

<p>5 is positive. Leave the \geq sign alone.</p> $\frac{5x}{5} \geq \frac{-30}{5}$ $x \geq -6$	<p>-4 is negative. Switch $<$ to $>$.</p> $\frac{x}{-4} < 6$ $-4 \cdot \left(\frac{x}{-4}\right) > (6) \cdot (-4)$ $x > -24$	$5 \cdot \frac{x}{5} < 7.5$ $x < 35$
$\frac{-9x}{-9} \leq \frac{72}{-9}$ $x \geq -8$	$-6 \cdot \frac{x}{6} < 8 \cdot -6$ $x > -48$	$3 \cdot \frac{x}{3} \geq -4 \cdot 3$ $x \geq -12$
$-10 \cdot \frac{x}{-10} \geq -3 \cdot -10$ $x \leq 30$	<p>$-x = - x$</p> $\frac{-x}{-1} > \frac{4}{-1}$ $x < -4$	$\frac{11x}{11} < \frac{-33}{11}$ $x < -3$
$-5 \cdot \frac{x}{-5} \leq 12 \cdot -5$ $x \geq -60$	$\frac{2x}{2} > \frac{-5}{2}$ $x > -\frac{5}{2}$	$\frac{-3x}{-3} < \frac{0}{-3}$ $x > 0$

Solving each inequality below takes more than one step. Remember to switch the inequality sign whenever you multiply or divide both sides by a negative number.

Adding -5 to each side is O.K.

$$-3x + 5 > 22$$

$$-3x > 17$$

$$-3x < \frac{17}{-3}$$

$$x < \frac{-17}{3}$$

Dividing by -3 means I have to reverse the sign.

$$5x + 1 \leq -44$$

$$\frac{5x}{5} \leq \frac{-45}{5}$$

$$x \leq -9$$

$$-2x + 15 < 7$$

$$\frac{-2x}{-2} < \frac{-8}{-2}$$

$$x > 4$$

$$2. \frac{x+15}{2} < -5.2$$

$$\frac{x+15}{-15} < \frac{-10}{-15}$$

$$x < -25$$

$$-3 \cdot \frac{x+9}{-3} > 3 \cdot -3$$

$$\frac{x+9}{-9} < \frac{-9}{-9}$$

$$x < -18$$

$$-10 \cdot \frac{x-6}{-10} \geq -4 \cdot -10$$

$$\frac{x-6}{+6} \leq \frac{40}{+6}$$

$$x \leq 46$$

$$4. \frac{-5x}{4} \geq 10.4$$

$$\frac{-5x}{-5} \geq \frac{40}{-5}$$

$$x \leq -8$$

$$7. \frac{3x}{7} < 1.7$$

$$\frac{3x}{3} < \frac{7}{3}$$

$$x < \frac{7}{3}$$

$$3. \frac{-4x}{3} > -12.3$$

$$\frac{-4x}{-4} > \frac{-36}{-4}$$

$$x < 9$$

$$\frac{x}{6} + 14 > 9$$

$$6. \frac{x}{6} > -5.6$$

$$x > -30$$

$$\frac{x}{-7} + 5 \leq 12$$

$$-7 \cdot \frac{x}{-7} \leq 7 \cdot -7$$

$$x \geq -49$$

$$\frac{x}{2} - 10 < -19$$

$$2. \frac{x}{2} < -9.2$$

$$x < -18$$

$$2x - 7x \leq 3$$

$$\frac{-5x}{-5} \leq \frac{3}{-5}$$

$$x \geq -\frac{3}{5}$$

$$\frac{-3(x-5)}{-3} > \frac{21}{-3}$$

$$\frac{x-5}{+5} < \frac{-7}{+5}$$

$$x < -2$$

$$\frac{9-x}{-9} \leq \frac{7}{-9}$$

$$\frac{-x}{-1} \leq \frac{-2}{-1}$$

$$x \geq 2$$

Here's how Sandy and Terry solved the last inequality on page 21.

Sandy	Terry
$9 - \cancel{x} \leq 7 + \cancel{x}$	$\cancel{9} - x \leq 7 - \cancel{9}$
$9 - 7 \leq \cancel{7} + x$	$-x \leq -2$
$2 \leq x$	$-1x \leq -2$
$x \geq 2$	$\frac{-1x}{-1} \geq \frac{-2}{-1}$
	$x \geq 2$

Both Sandy and Terry ended up with the same solution set. Whose method do you like better? Why?

Both methods work. Use either one to solve each inequality below.

$2 - x > 16$ $\begin{array}{r} -2 \qquad -2 \\ -x > 14 \\ \hline -1 \quad -1 \\ x < -14 \end{array}$	$25 \leq 10 - x$ $\begin{array}{r} -10 \quad -10 \\ 15 \leq -x \\ \hline -1 \quad -1 \\ -15 \geq x \text{ or} \\ x \leq -15 \end{array}$	$-5x + 9 \geq 3x$ $\begin{array}{r} +5x \qquad +5x \\ 9 \geq 8x \\ \hline \frac{9}{8} \geq \frac{8x}{8} \\ \frac{9}{8} \geq x \text{ or } 1\frac{1}{8} \geq x \\ x \leq 1\frac{1}{8} \end{array}$
$3x + 15 > 3 + 7x$ $\begin{array}{r} -3x \qquad -3x \\ 15 > 3 + 4x \\ \hline -3 \quad -3 \\ 12 > 4x \\ \hline \frac{12}{4} > \frac{4x}{4} \\ 3 > x \text{ or } x < 3 \end{array}$	$x + 20 < 5x - 8$ $\begin{array}{r} -x \qquad -x \\ 20 < 4x - 8 \\ \hline +8 \quad +8 \\ 28 < 4x \\ \hline \frac{28}{4} < \frac{4x}{4} \\ 7 < x \text{ or } x > 7 \end{array}$	$18 + x \geq 12 + 7x$ $\begin{array}{r} -x \qquad -x \\ 18 \geq 12 + 6x \\ \hline -12 \quad -12 \\ 6 \geq 6x \\ \hline \frac{6}{6} \geq \frac{6x}{6} \\ 1 \geq x \text{ or } x \leq 1 \end{array}$
$\frac{5(3 - x)}{5} < \frac{50}{5}$ $\begin{array}{r} 3 - x < 10 \\ \hline -3 \quad -3 \\ -x < 7 \\ \hline -1 \quad -1 \\ x > -7 \end{array}$	$2 \cdot \frac{4 - x}{2} > 12 \cdot 2$ $\begin{array}{r} 4 - x > 24 \\ \hline -4 \quad -4 \\ -x > 20 \\ \hline -1 \quad -1 \\ x < -20 \end{array}$	$12 - 7x \leq 12$ $\begin{array}{r} -12 \qquad -12 \\ -7x \leq 0 \\ \hline -7 \quad -7 \\ x \geq 0 \end{array}$

Solve each inequality and graph the solution set.

$$3x - 4x > 6$$

$$\begin{array}{r} -x > 6 \\ \hline -1 & \quad \hline -1 & \quad \hline x < -6 \end{array}$$



$$\begin{array}{r} x + 12 \geq 2x - 5 \\ -x + 5 \quad -x + 5 \end{array}$$

$$\begin{array}{r} 17 \geq x \\ x \leq 17 \end{array}$$



$$2. \frac{x-7}{2} < -6 \cdot 2$$

$$\begin{array}{r} x-7 < -12 \\ +7 \quad +7 \\ x < -5 \end{array}$$



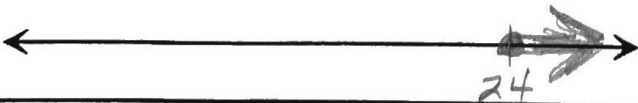
$$\frac{x}{2} - 7 < -6$$

$$\begin{array}{r} \frac{x}{2} - 7 < -6 \\ +7 \quad +7 \\ 2. \frac{x}{2} < 1 \cdot 2 \\ x < 2 \end{array}$$



$$7x - 2x - x \geq 24 + 3x$$

$$\begin{array}{r} 4x \geq 24 + 3x \\ -3x \quad -3x \\ x \geq 24 \end{array}$$



$$\frac{4x}{3} + 1 < 4$$

$$\begin{array}{r} 3. \frac{4x}{3} < 3 \cdot 3 \\ \frac{4x}{4} < \frac{9}{4} \quad x < 2\frac{1}{4} \end{array}$$



$$\begin{array}{r} -16 + 4x > 10 - x \\ +16 \quad +x \quad +16 \quad +x \end{array}$$

$$\begin{array}{r} 5x > 26 \\ \hline 5 & \quad \hline 5 & \quad \hline x > \frac{26}{5} \end{array}$$



$$3(x-4) - 9x \geq 2x - 4$$

$$\begin{array}{r} 3x - 12 - 9x \geq 2x - 4 \\ -6x - 12 \geq 2x - 4 \\ +6x \quad +4 \quad +6x \quad +4 \end{array}$$

$$\begin{array}{r} -8 \geq 8x \\ \hline -8 & \quad \hline -8 & \quad \hline -1 \geq x \text{ or } \\ x \leq -1 \end{array}$$



When an absolute value sign appears in an equation or inequality, you should *not* use the Addition, Multiplication or Division Principle to simplify an expression *inside* the absolute value sign. Instead, think about what the absolute value sign means.

The number inside the $| |$ sign could be either 6 or -6.

$$|x + 1| = 6$$

$$x + 1 = 6 \text{ or } x + 1 = -6$$

$$x = 5 \text{ or } x = -7$$

To make sure that both 5 and -7 are solutions, we can substitute each for x .

Check: $|5 + 1| = |6| = 6$
 $|-7 + 1| = |-6| = 6$

Solve each equation, and check your solutions.

$$|x + 5| = 7$$

$$x + 5 = 7 \quad x + 5 = -7$$

$$\begin{matrix} -5 & -5 \\ \hline x = 2 & \text{or } x = -12 \end{matrix}$$

Check: $|2 + 5| = 7$ $|-12 + 5| = 7$
 $|7| = 7$ $|-7| = 7$ ✓

$$|x - 10| = 2$$

$$x - 10 = 2 \quad x - 10 = -2$$

$$\begin{matrix} +10 & +10 & +10 & +10 \\ \hline x = 12 & \text{or } x = 8 \end{matrix}$$

Check: $|12 - 10| = 2$ $|8 - 10| = 2$
 $|2| = 2$ $|-2| = 2$ ✓

$$\left| \frac{x}{5} \right| = 2$$

$$5 \cdot \frac{x}{5} = 2 \cdot 5 \quad 5 \cdot \frac{x}{5} = -2 \cdot 5$$

$$x = 10 \quad \text{or} \quad x = -10$$

Check: $\left| \frac{10}{5} \right| = 2$ $\left| \frac{-10}{5} \right| = 2$ ✓

$$| -3x | = 4$$

$$-3x = 4 \quad -3x = -4$$

$$\begin{matrix} -3 & -3 & -3 & -3 \\ \hline x = -4/3 & \text{or } x = 4/3 \end{matrix}$$

Check: $| -3(-4/3) | = 4$ $| -3(4/3) | = 4$ ✓

$$|3x + 4| = 10$$

$$3x + 4 = 10 \quad 3x + 4 = -10$$

$$\begin{matrix} -4 & -4 \\ \hline 3x = 6 & 3x = -14 \\ \hline x = 2 & \text{or } x = -14/3 \end{matrix}$$

Check: $|3(2) + 4| = 10$ $|3(-14/3) + 4| = 10$
 $|6 + 4| = 10$ $|-14 + 4| = 10$

$$|12x - 6| = 6$$

$$12x - 6 = 6 \quad 12x - 6 = -6$$

$$\begin{matrix} +6 & +6 & +6 & +6 \\ \hline 12x = 12 & 12x = 0 \\ \hline x = 1 & \text{or } x = 0 \end{matrix}$$

Check: $|12(1) - 6| = 6$ $|12(0) - 6| = 6$
 $|6| = 6$ $|-6| = 6$ ✓

24

$$|10| = 10$$

$$|-14 + 4| = 10$$

$$|-10| = 10$$

$$|12 - 6| = 6$$

$$|6| = 6$$

$$|-6| = 6$$

Solve each equation.

The 3 is outside the | | sign, so I can use the Addition Principle.

$$|x| + 3 = 20$$

$$|x| = 17$$

$$x = 17 \text{ or } -17$$

$$|x| - 8 = -2$$

$$|x| = 6$$

$$x = 6 \text{ or } x = -6$$

$$|x| + 8 = 11$$

$$|x| = 3$$

$$x = 3 \text{ or } x = -3$$

$$2|x| - 5 = 3$$

$$\frac{2|x|}{2} = \frac{8}{2}$$

$$|x| = 4$$

$$x = 4 \text{ or } -4$$

$$|4x - 5| = 7$$

$$4x - 5 = 7$$

$$\frac{4x}{4} = \frac{12}{4}$$

$$x = 3 \text{ or } x = -\frac{1}{2}$$

$$4x - 5 = -7$$

$$\frac{4x}{4} = \frac{-2}{4}$$

$$4|x| - 5 = 7$$

$$\frac{4|x|}{4} = \frac{12}{4}$$

$$|x| = 3$$

$$x = 3 \text{ or } -3$$

$$|x + 2| - 3 = 7$$

$$|x + 2| = 10$$

$$x + 2 = 10$$

$$x = 8$$

$$x + 2 = -10$$

$$x = -12$$

$$x = 8 \text{ or } x = -12$$

$$|x + 5| + 4 = 12$$

$$|x + 5| = 8$$

$$x + 5 = 8$$

$$x = 3$$

$$x + 5 = -8$$

$$x = -13$$

$$x = 3 \text{ or } x = -13$$

$$|2x - 3| + 2 = 11$$

$$|2x - 3| = 9$$

$$2x - 3 = 9$$

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3 \text{ or } x = -3$$

$$2x - 3 = -9$$

$$\frac{2x}{2} = \frac{-6}{2}$$

$$-3|x| + 5 = -4$$

$$\frac{-3|x|}{-3} = \frac{-9}{-3}$$

$$|x| = 3$$

$$x = 3 \text{ or } -3$$

Solve and check each inequality.

You can't check all the solutions, so pick a few samples.

Numbers from 9 down to -9 will work.

$$|x| + 2 < 11$$

$$|x| < 9$$

$$x < 9 \text{ and } x > -9$$

Check:

$$6: |6| + 2 = 6 + 2 = 8 < 11$$

$$-4: |-4| + 2 = 4 + 2 = 6 < 11$$

What's inside must be over 3 or under -3.

$$|x - 2| > 3$$

$$x - 2 > 3 \text{ or } x - 2 < -3$$

$$x > 5 \text{ or } x < -1$$

Check:

$$9: |9 - 2| = 7 > 3$$

$$-4: |-4 - 2| = |-6| = 6 > 3$$

$$|x| > 12$$

$$x < -12 \text{ and } x > 12$$

Check:

$$3 < 12$$

$$+2 > -12$$

$$|x| \leq 4$$

$$x \leq 4 \text{ and } x \geq -4$$

Check:

$$2 \leq 4$$

$$\text{and } 0 \geq -4$$

$$|x| + 4 < 20$$

$$|x| < 16$$

$$x < 16 \text{ and } x > -16$$

Check:

$$5 < 16$$

$$-1 > -16$$

$$|x| - 6 > 1$$

$$|x| > 7$$

$$x > 7 \text{ or } x < -7$$

Check:

$$10 > 7$$

$$0 < -7$$

$$|x - 1| < 5$$

$$x - 1 < 5 \quad x - 1 > -5$$

$$x < 6 \quad x > -4$$

Check:

$$3 < 6$$

$$3 > -4$$

$$|x + 10| > 5$$

$$x + 10 > 5 \quad x + 10 < -5$$

$$x > -5 \quad x < -15$$

Check:

$$0 > -5$$

$$-20 < -15$$

$$|12x| \geq 24$$

$$\frac{12x}{12} \geq \frac{24}{12} \quad \frac{12x}{12} \leq \frac{-24}{12}$$

$$x \geq 2 \quad x \leq -2$$

Check:

$$10 \geq 2$$

$$-5 \leq -2$$

$$\left| \frac{x}{3} \right| < 8$$

$$3 \cdot \frac{x}{3} < 8 \cdot 3 \quad 3 \cdot \frac{x}{3} > -18 \cdot 3$$

$$x < 24 \quad x > -54$$

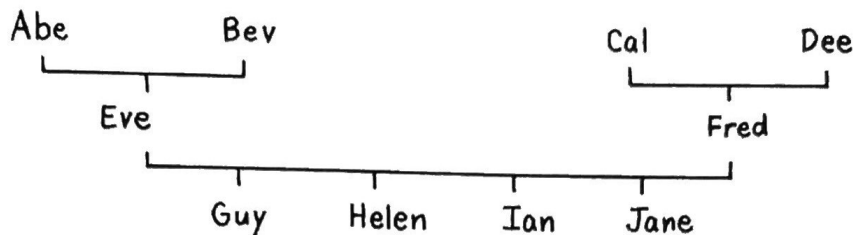
Check:

$$5 < 24$$

$$-3 > -54$$

Relations

In algebra, =, <, >, ≤ and ≥ are sometimes called **relations**. Thinking of relations in your family can help you understand relations in algebra. Relations in your family are people connected to you in certain ways. You have different kinds of relations: mother, father, brothers, sisters, aunts, uncles, etc. Look at this family tree:



The tree shows Abe is married to Bev, and Eve is their child. Cal is married to Dee, and their child is Fred. Guy, Helen, Ian and Jane are the children of Eve and Fred.

Let's see how some of the people in this family are related:

Sisters: We will call the sister relation S . $S(x)$ means "a sister of x ."

$S(\text{Guy}) = \text{Helen}$	means	<u>"A sister of Guy is Helen."</u>
$S(\text{Guy}) = \text{Jane}$	means	<u>"A sister of Guy is Jane."</u>
$S(\text{Ian}) = \text{Helen}$	means	<u>A sister of Ian is Helen</u>
$S(\text{Ian}) = \text{Jane}$	means	<u>A sister of Ian is Jane</u>
$S(\text{Jane}) = \text{Helen}$	means	<u>A sister of Jane is Helen</u>
$S(\text{Helen}) = \text{Jane}$	means	<u>A sister of Helen is Jane</u>

Brothers: We will call this relation B . $B(x)$ means "a brother of x ."

$B(\text{Ian}) = \text{Guy}$	$B(\text{Jane}) = \text{Guy}$	$B(\text{Jane}) = \text{Ian}$
$B(\text{Guy}) = \text{Ian}$	$B(\text{Helen}) = \text{Guy}$	$B(\text{Helen}) = \text{Ian}$

Fathers: We will call this relation F . $F(x)$ means "the father of x ."

$F(\text{Eve}) = \text{Abe}$	$F(\text{Ian}) = \text{Fred}$	$F(\text{Guy}) = \text{Fred}$
$F(\text{Fred}) = \text{Cal}$	$F(\text{Helen}) = \text{Fred}$	$F(\text{Jane}) = \text{Fred}$

Grandmothers: We will call this relation G . $G(x)$ means "a grandmother of x ."

$G(\text{Helen}) = \text{Bev}$	$G(\text{Jane}) = \text{Bev}$	$G(\text{Ian}) = \text{Bev}$
$G(\text{Helen}) = \text{Dee}$	$G(\text{Jane}) = \text{Dee}$	$G(\text{Ian}) = \text{Dee}$
$G(\text{Guy}) = \text{Bev}$	$G(\text{Guy}) = \text{Dee}$	

Relations in families pair people with other people. Relations in algebra pair numbers with other numbers. Here are some relations which involve numbers:

The "greater than" relation: We will use $G(x)$ to mean "a number greater than x ."

$$G(5) = 7 \quad \text{means} \quad \underline{\text{"A number greater than 5 is 7." or } 7 > 5}$$

$$G(5) = 10.4 \quad \text{means} \quad \underline{\text{"A number greater than 5 is 10.4." or } 10.4 > 5}$$

Name some other numbers which can be paired up with 5 in this relation:

$$G(5) = 10$$

$$G(5) = 19$$

$$G(5) = 26$$

$$G(5) = 72$$

Find a number to make each of these true:

$$G(2) = 3$$

$$G(-6) = 0$$

$$G(100) = 102$$

$$G(4.8) = 5$$

The "less than" relation: We will use $L(x)$ to mean "a number less than x ."

$$L(3) = 2 \quad \text{means} \quad \underline{\text{"A number less than 3 is 2." or } 2 < 3}$$

$$L(3) = -\frac{1}{3} \quad \text{means} \quad \underline{\text{"A number less than 3 is } -\frac{1}{3}\text{." or } -\frac{1}{3} < 3}$$

$$L(-1) = -3 \quad \text{means} \quad \underline{A \# \text{ less than } -1 \text{ is } -3 \quad -3 < -1}$$

$$L(-2.5) = -7 \quad \text{means} \quad \underline{A \# \text{ less than } -2.5 \text{ is } -7 \quad -7 < -2.5}$$

The "equality" relation: We will use $E(x)$ to mean "a number equal to x ."

$$E(2) = 2 \quad \text{means} \quad \underline{\text{"A number equal to 2 is 2." or } 2 = 2}$$

$$E(-6) = -6 \quad \text{means} \quad \underline{A \# \text{ equal to } -6 \text{ is } -6 \quad -6 = -6}$$

$$E(11.2) = 11.2 \quad \text{means} \quad \underline{A \# \text{ equal to } 11.2 \text{ is } 11.2 \quad 11.2 = 11.2}$$

The "less than or equal to" relation: We will use $T(x)$ to mean "a number less than or equal to x ."

$$T(4) = 0 \quad \text{means} \quad \underline{\text{"A number less than or equal to 4 is 0." or } 0 \leq 4}$$

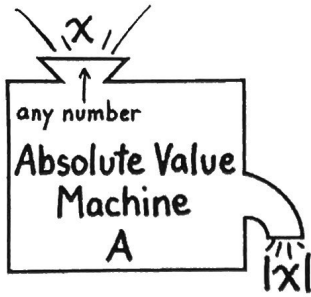
$$T\left(\frac{3}{5}\right) = \frac{3}{5} \quad \text{means} \quad \underline{A \# \text{ less than or equal to } \frac{3}{5} \text{ is } \frac{3}{5} \quad \frac{3}{5} \leq \frac{3}{5}}$$

$$T(-6.3) = -7 \quad \text{means} \quad \underline{A \# \text{ less than or equal to } -6.3 \text{ is } -7}$$

$$-7 \leq -6.3$$

Here are some more examples of functions.

The "absolute value" function: $A(x)$ will mean "the absolute value of x ." $A(x) = |x|$



$$A(7) = |7| = 7$$

$$A(-5.3) = |-5.3| = 5.3$$

$$A(-7) = |-7| = 7$$

$$A(4.71) = |4.71| = 4.71$$

$$A(3) = |3| = 3$$

$$A\left(\frac{1}{2}\right) = \left|\frac{1}{2}\right| = \frac{1}{2}$$

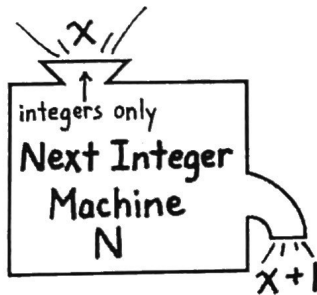
$$A(-3) = |-3| = 3$$

$$A\left(\frac{-3}{4}\right) = \left|\frac{-3}{4}\right| = \frac{3}{4}$$

$$A(-12) = |-12| = 12$$

$$A(-6.6) = |-6.6| = 6.6$$

The "next integer" function: $N(x)$ will mean "the integer after x ." $N(x) = x + 1$



$$N(10) = 11$$

$$N(-16) = -15$$

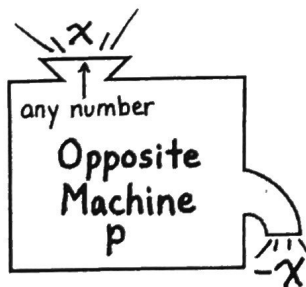
$$N(9) = 10$$

$$N(-101) = -100$$

$$N(0) = 1$$

$$N(25) = 26$$

The "opposite" function: $P(x)$ will mean "the opposite of x ." $P(x) = -x$



$$P(2) = -2$$

$$P\left(\frac{3}{5}\right) = -\frac{3}{5}$$

$$P(3.5) = -3.5$$

$$P(-1) = 1$$

$$P\left(\frac{-1}{2}\right) = \frac{1}{2}$$

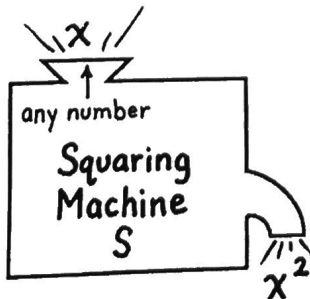
$$P(0.6) = -0.6$$

$$P(0) = 0$$

$$P\left(\frac{-10}{7}\right) = \frac{10}{7}$$

$$P(-0.003) = 0.003$$

The "squaring" function: $S(x)$ will mean "the square of x ." $S(x) = x^2$



$$S(3) = 9$$

$$S(-3) = 9$$

$$S(12) = 144$$

$$S(4) = 16$$

$$S(-4) = 16$$

$$S(100) = 10,000$$

$$S(5) = 25$$

$$S(-5) = 25$$

$$S(-100) = 10,000$$

$$S(6) = 36$$

$$S(-6) = 36$$

$$S(0.5) = 0.25$$

Here are some functions we use in everyday life:

The "first class postage" function: $P(x)$ is the first class postage on a letter weighing x ounces. It costs 25¢ to mail a letter weighing one ounce or less. For each additional ounce or part of an ounce you pay 20¢ more.

round to the whole #
 $P(x) = 20x + .25$

$$P(1) = .25$$

$$P\left(\frac{1}{4}\right) = .25$$

$$P(0.7) = .25$$

$$P(2) = .45$$

$$P\left(\frac{3}{4}\right) = .25$$

$$P(7) = 1.45$$

$$P(3) = .65$$

$$P\left(1\frac{1}{4}\right) = .45$$

$$P(7.3) = 1.65$$

$$P(4) = .85$$

$$P\left(2\frac{1}{4}\right) = .65$$

$$P(3.7) = .85$$

$$P(5) = 1.05$$

$$P\left(3\frac{1}{2}\right) = .85$$

$$P(4.9) = 1.05$$

The "feet-to-inches" function: $I(x)$ is the number of inches in x feet. There are 12 inches in one foot, so $I(1) = 12$.

$$I(x) = 12x$$

$$I(2) = 24$$

$$I(5) = 60$$

$$I\left(\frac{1}{2}\right) = 6$$

$$I(3) = 36$$

$$I(9) = 108$$

$$I\left(1\frac{1}{2}\right) = 18$$

$$I(4) = 48$$

$$I(15) = 180$$

$$I\left(\frac{1}{3}\right) = 4$$

The "sales tax" function: $S(x)$ is the sales tax on a taxable purchase of x dollars.

To figure these out, you need to know the sales tax percentage for your state. Try to get a copy of the sales tax table which many stores use.

$$S(x) = 7.25\%x$$

$$S(.10) = .0725(.10) = 0.00725$$

$$S(.50) = 0.03625$$

$$S(.87) = 0.063075$$

$$S(.20) = 0.0145$$

$$S(1.00) = .0725$$

$$S(1.10) = 0.07975$$

$$S(.30) = 0.02175$$

$$S(2.00) = 0.145$$

$$S(7.75) = 0.561875$$

The "days-in-a-year" function: $D(x)$ is the number of days in the year x .

Leap years have 366 days. Others have 365 days.

$$D(1980) = 366$$

$$D(1971) = 365$$

$$D(1776) = 366$$

$$D(1950) = 365$$

$$D(1900) = 366$$

$$D(2000) = 366$$

Instead of describing a function in words, we often define it by giving an algebraic expression for the number which is paired up with x . You finish this example:

The "f" function: $f(x)$ is one more than 3 times x . $f(x) = 3x + 1$

$$f(5) = 3(5) + 1 = 15 + 1 = 16 \quad f(8) =$$

$$f(-2) = 3(-2) + 1 = -6 + 1 = -5 \quad f(-8) =$$

Each function below is defined by an algebraic expression. Find the numbers asked for by substituting in the expression.

$$g(x) = 3x - 1$$

$$g(-4) = 3(-4) - 1 = -12 - 1 = -13$$

$$g(7) = 3(7) - 1 = 21 - 1 = 20$$

$$g(5) = 3(5) - 1 = 15 - 1 = 14$$

$$h(x) = 3(x - 1)$$

$$h(-4) = 3(-4 - 1) = 3(-5) = -15$$

$$h(7) = 3(7 - 1) = 3(6) = 18$$

$$h(5) = 3(5 - 1) = 3(4) = 12$$

$$p(x) = x^2 + 4$$

$$p(3) = 3^2 + 4 = 9 + 4 = 13$$

$$p(1) = 1^2 + 4 = 1 + 4 = 5$$

$$p(-10) = (-10)^2 + 4 = 100 + 4 = 104$$

$$q(x) = (x + 4)^2$$

$$q(3) = (3 + 4)^2 = 7^2 = 49$$

$$q(1) = (1 + 4)^2 = 5^2 = 25$$

$$q(-10) = (-10 + 4)^2 = (-6)^2 = 36$$

$$r(x) = |x + 6|$$

$$r(3) = |3 + 6| = |9| = 9$$

$$r(-10) = |-10 + 6| = |-4| = 4$$

$$r(-6) = |-6 + 6| = |0| = 0$$

$$s(x) = |x| + 6$$

$$s(3) = |3| + 6 = 3 + 6 = 9$$

$$s(-10) = |-10| + 6 = 10 + 6 = 16$$

$$s(-6) = |-6| + 6 = 6 + 6 = 12$$

$$m(x) = x - 1$$

$$m(8) = 8 - 1 = 7$$

$$m(-8) = -8 - 1 = -9$$

$$m(0) = 0 - 1 = -1$$

$$n(x) = 1 - x$$

$$n(8) = 1 - 8 = -7$$

$$n(-8) = 1 - (-8) = 9$$

$$n(0) = 1 - 0 = 1$$

A **table** is an easy way to list the pairs of numbers that belong to a function. The expression for the function is written above the table. The first column of the table lists the numbers to be substituted for the variable in the function. The second column lists the **values** obtained by substituting the numbers in the first column.

$$f(x) = x - 10$$

x	f(x)
25	$25 - 10 = 15$
17	$17 - 10 = 7$
12	$12 - 10 = 2$
10	$10 - 10 = 0$
8	$8 - 10 = -2$
2	$2 - 10 = -8$
0	$0 - 10 = -10$
-3	$-3 - 10 = -13$

$$g(x) = 2x + 6$$

x	g(x)
-6	$2(-6) + 6 = -6$
-2	$2(-2) + 6 = 2$
-1	$2(-1) + 6 = 4$
0	$2(0) + 6 = 6$
1	$2(1) + 6 = 8$
2	$2(2) + 6 = 10$
6	$2(6) + 6 = 18$
100	$2(100) + 6 = 206$

$$h(x) = 0 \cdot x$$

x	h(x)
-90	$0(-90) = 0$
-10	$0(-10) = 0$
-2	$0(-2) = 0$
-1	$0(-1) = 0$
0	$0(0) = 0$
1	$0(1) = 0$
15	$0(15) = 0$
40	$0(40) = 0$

$$k(x) = \frac{x}{6}$$

x	k(x)
0	$\frac{0}{6} = 0$
18	$\frac{18}{6} = 3$
-18	$\frac{-18}{6} = -3$
60	$\frac{60}{6} = 10$
-60	$\frac{-60}{6} = -10$
1	$\frac{1}{6}$
-1	$\frac{-1}{6}$
5	$\frac{5}{6}$

$$m(x) = \frac{3x}{2}$$

x	m(x)
-3	$\frac{3(-3)}{2} = \frac{-9}{2}$
-2	$\frac{3(-2)}{2} = \frac{-6}{2} = -3$
-1	$\frac{3(-1)}{2} = \frac{-3}{2}$
0	$\frac{3(0)}{2} = \frac{0}{2} = 0$
1	$\frac{3(1)}{2} = \frac{3}{2}$
2	$\frac{3(2)}{2} = \frac{6}{2} = 3$
3	$\frac{3(3)}{2} = \frac{9}{2}$
4	$\frac{3(4)}{2} = \frac{12}{2} = 6$

$$n(x) = \frac{x-5}{4}$$

x	n(x)
-3	$\frac{(-3)-5}{4} = \frac{-8}{4} = -2$
-2	$\frac{(-2)-5}{4} = \frac{-7}{4}$
-1	$\frac{(-1)-5}{4} = \frac{-6}{4} = \frac{-3}{2}$
0	$\frac{(0)-5}{4} = \frac{-5}{4}$
1	$\frac{(1)-5}{4} = \frac{-4}{4} = -1$
2	$\frac{(2)-5}{4} = \frac{-3}{4}$
3	$\frac{(3)-5}{4} = \frac{-2}{4} = \frac{-1}{2}$
4	$\frac{(4)-5}{4} = \frac{-1}{4}$

Many functions important in science and other fields have been discovered by people who collected data, organized their information in a table, and figured out a **formula** for it.

See if you can find a formula to fit the data in each table.

$$f(x) = 3x$$

x	f(x)
5	15
2	6
1	3
0	0
-1	-3
-2	-6
-5	-15

$$g(x) = x + 2$$

x	g(x)
10	12
5	7
3	5
0	2
-3	-1
-5	-3
-15	-13

$$h(x) = x - 4$$

x	h(x)
8	4
4	0
2	-2
0	-4
-2	-6
-4	-8
-8	-12

$$p(x) = -x$$

x	p(x)
-20	20
-12	12
-6	6
0	0
6	-6
12	-12
20	-20

$$q(x) = 2x + 1$$

x	q(x)
4	9
2	5
1	3
0	1
-1	-1
-2	-3
-4	-7

$$r(x) = x^2$$

r	r(x)
3	9
2	4
1	1
0	0
-1	1
-2	4
-3	9

$$s(x) = -2x$$

x	s(x)
5	-10
3	-6
1	-2
-1	2
-3	6
-5	10
-7	14

$$v(x) = |x|$$

v	v(x)
7	7
4	4
0	0
-2	2
-5	5
-8	8
-60	60