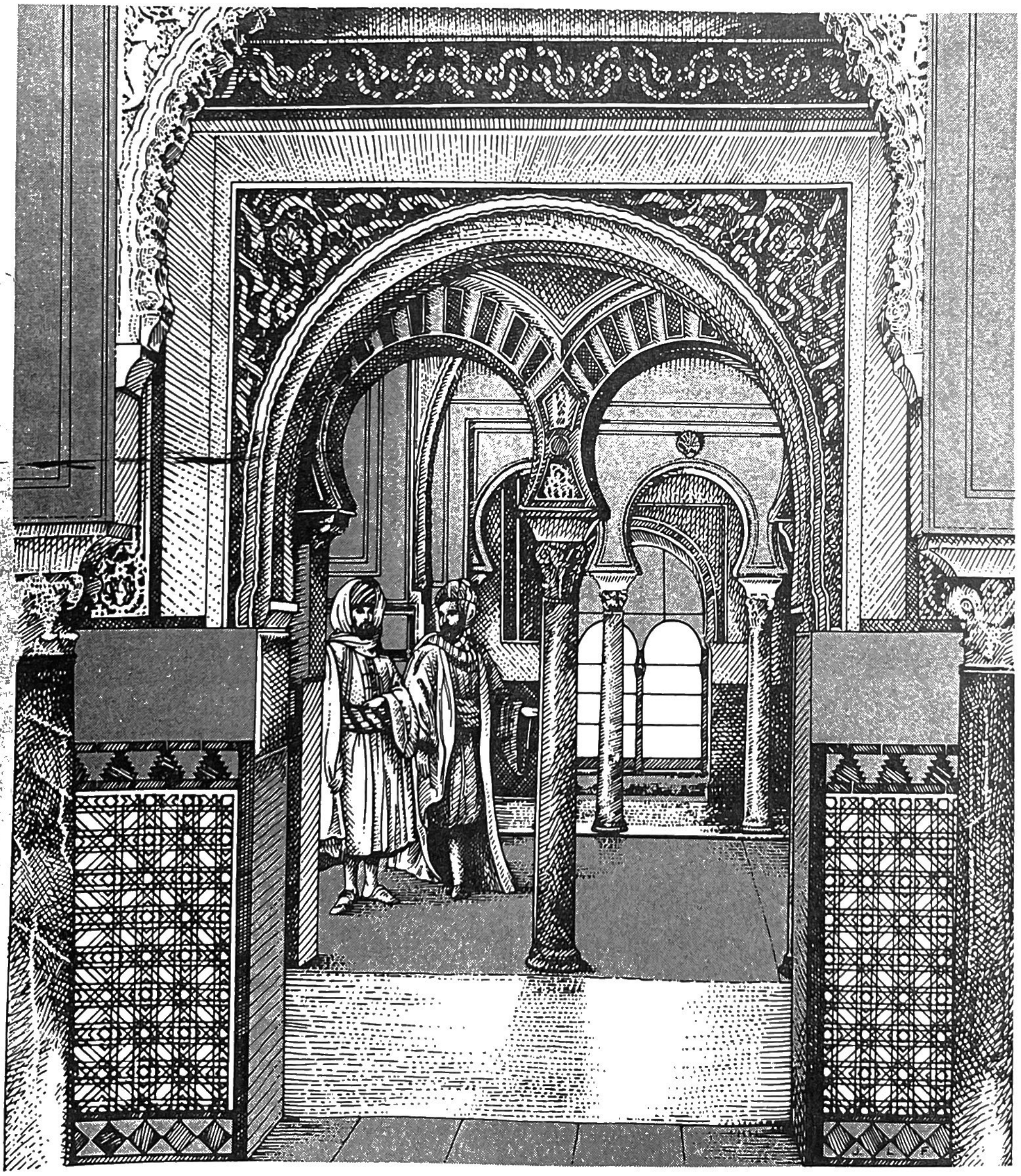
Key to

Algebra®

2

Student Workbook

Variables, Terms, and Expressions



By Julie King and Peter Rasmussen

Name Class

TABLE OF CONTENTS

Al-Khowarizmi

One of the first great institutes of learning in the world was established at Alexandria in Egypt about 300 B.C. It was simply called the Museum. It housed such great mathematicians as Euclid, Eratosthenes and Hypatia.

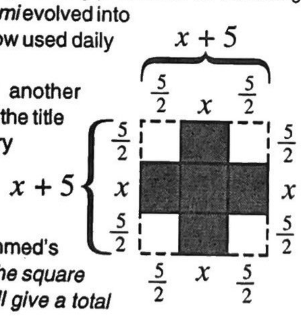
There were several other similar institutes in the western world, including Plato's Academy in Athens, but they were all closed by official decree in the year 529 A.D. Scholars began to look to the East for a place to carry on their studies, and they found one in the "House of Wisdom" at Baghdad in Iraq.

Among the faculty in the year 800 at the House of Wisdom was the Arab mathematician Mohammed ibn-Musa al-Khowarizmi. His name tells us that he was called Mohammed, his father was called Musa, and he hailed from the city of Khowarizmi.

Mohammed wrote a book called *Concerning the Hindu Art of Reckoning* in which he introduced the Hindu decimal number system. His explanations were so clear, and the operations on this system were so easy to carry out, that the whole Arabic civilization adopted it at once. It was introduced into Europe during the Renaissance, and it is the system used all over the world today. We call it the Hindu-Arabic system in honor of its creation by the Hindus and transmission by the Arabs.

When European readers of Mohammed's book read it they associated his Latin name, or *algorismi*, with any procedure for obtaining a definite answer to a problem. *Algorismi* evolved into the English word *algorithm*, which is now used daily in computer science.

Mohammed is responsible for another common English word. The first part of the title of his most important book is *Al-jabr*. Try to sound it out in English. Does it sound like the word *algebra*? It should.



Much of what is studied in algebra today is contained in Mohammed's *Al-jabr*. One problem reads: *What is the square which combined with 10 of its roots will give a total of 39?*

Today we write this as the equation $x^2 + 10x = 39$.

Historical note by David Zitarelli

Illustration by Jay Flom

Mohammed solved the problem by drawing one square, putting a second square around it, and using the second square to find the length of a side of the first square. He found that the desired length was 3. Notice that $3^2 + (10)(3) = 39$. Today we call the algorithm that Mohammed used "the method of completing the square."

Mohammed was the best mathematician of the Middle Ages. His *Al-jabr* solved inheritance problems to illustrate the usefulness of algebra. It remained the best mathematics book in the world for over 700 years. The western world owes a debt of gratitude to this Arabic scientist not only for his contributions to mathematics but for his work in astronomy as well.

On the cover of this book Mohammed ibn-Musa al-Khowarizmi speaks with one of his students inside the library of a mosque.

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Review of Operations on Integers

In Book 1 you were told about numbers called integers — positive integers, negative integers and zero. You learned how to add, subtract, multiply and divide integers.

Adding Integers: Think of positive integers as gains, negative integers as losses and zero as showing no change. To add two or more integers, just think of the integer which shows the overall change.

Subtracting Integers: Think of the problem as an adding problem — only be sure to add the opposite of the number you are subtracting.

Multiplying and Dividing Integers: Break the problem down into two parts. Find the amount by multiplying or dividing. The sign will be positive if you are multiplying or dividing two numbers with the same sign. The sign will be negative if the two numbers you are multiplying or dividing have different signs.

Below are some problems for you to do. Some are adding, some are subtracting, some are multiplying and some are dividing — so *be careful* . . .

${}^{-}5 + {}^{-}3 =$	${}^{-}6 \cdot {}^{-}9 =$	${}^{-}8 - 8 =$	${}^{-}8 \cdot 1 =$
${}^{-}8 \cdot 5 =$	$4 + {}^{-}14 =$	${}^{-}5 - {}^{-}5 =$	${}^{-}10 \cdot 0 =$
$5 \cdot {}^{-}7 =$	${}^{-}24 \div 3 =$	${}^{-}18 \div {}^{-}9 =$	${}^{-}3 - 3 =$
${}^{-}6 - 4 =$	${}^{-}5 \cdot {}^{-}5 =$	$4 \cdot 0 =$	$8 - 8 =$
$20 \div {}^{-}2 =$	$7 \cdot 7 =$	${}^{-}8 + 8 =$	$0 \div 6 =$
$10 + {}^{-}16 =$	$14 + {}^{-}14 =$	$1 \cdot 7 =$	$-({}^{-}3) =$
$(10)({}^{-}16) =$	$({}^{-}3)(3) =$	$0 \cdot {}^{-}3 =$	$-(5) =$
$5 - 14 =$	$14 - 6 =$	${}^{-}5 + {}^{-}5 =$	$-({}^{-}9) =$
${}^{-}1 \cdot 6 =$	$1 \cdot 1 =$	$({}^{-}8)({}^{-}1) =$	$-(0) =$

These problems take more than one step:

$5 \cdot 3 - 2 =$	$4 + (6 - 7) =$	${}^{-}8 + 3({}^{-}2) =$
$5 \cdot (3 - 2) =$	$4 + 6 - 7 =$	$({}^{-}8 + 3)({}^{-}2) =$

Variables and Expressions

Here are some problems that follow a pattern:

$3 + 6$
$^{-}10 + 6$
$^{-}5 + 6$
$8 + 6$
$2 + 6$
$6 + 6$
$37 + 6$

To write the pattern for a group of problems just copy down the part that is always the same and use a letter in place of each number that changes. Here is the pattern for the problems above:

$$x + 6$$

Letters that are used where numbers can go are called **variables**. Patterns are also called **expressions.** Here are some examples:

Variables: x a t b y

Expressions: $2 \cdot x$ $x + 4$ $3 \cdot x + 4$ $x - 5$ $x + y$ $3 \cdot (a + 6)$

We usually don't write the multiplication dots in expressions if the meaning is clear without them. For example,

$x \cdot y$ is written xy

$2 \cdot x$ is written $2x$

$3 \cdot x + 4$ is written $3x + 4$

$3 \cdot (a + 6)$ is written $3(a + 6)$

$a \cdot b$ is written ________

$6 \cdot x$ is written ________

$2 \cdot n + 5$ is written ________

$4 \cdot (x - 3)$ is written ________

Write an expression for each group of problems.

$3\cdot 5+2$	$8+4$	$5\cdot 2$	$3(5+1)$
$3\cdot 6+2$	$3+4$	$5\cdot 7$	$3(9+1)$
$3\cdot 8+2$	$0+4$	$5\cdot 4$	$3(2+1)$
$3\cdot {}^{-}4+2$	${}^{-}3+4$	$5\cdot 1$	$3(3+1)$
$3\cdot 3+2$	$4+4$	$5\cdot 13$	$3({}^{-}6+1)$
$3\cdot 2+2$	$13+4$	$5\cdot {}^{-}6$	$3(0+1)$
$3\cdot 10+2$	$10+4$	$5\cdot 8$	$3({}^{-}2+1)$
Expression: $3a+2$	Expression:	Expression:	Expression:

You make up some problems to go with these two expressions:

$5-2\cdot 8$	$-({}^{-}4)+3$		
$5-2\cdot 7$	$-(6)+3$		
$5-2\cdot 4$	$-({}^{-}8)+3$		
$5-2\cdot 0$	$-({}^{-}5)+3$		
$5-2\cdot {}^{-}3$	$-(2)+3$		
$5-2\cdot 10$	$-(15)+3$		
$5-2\cdot {}^{-}8$	$-({}^{-}6)+3$		
Expression:	Expression:	Expression: $y-7$	Expression: $6x+5$

This expression shows a pattern:

$$x + 10$$

In order to make up problems that follow this pattern we just **substitute** different numbers for the variable.

Substitute 3 for x: $3 + 10 = 13$

Substitute 7 for x: $7 + 10 = 17$

Substitute ⁻5 for x: $^-5 + 10 = 5$

You substitute the given numbers in each expression below.

Expression: $y - 4$

Substitute 7 for y: $7 - 4 = 3$

Substitute 5 for y: $5 - 4 =$

Substitute 4 for y:

Substitute 3 for y:

Expression: $5x + 2$

Substitute 4 for x: $5 \cdot 4 + 2 =$

Substitute 5 for x:

Substitute ⁻5 for x:

Substitute 0 for x:

Expression: $3x$

Substitute 5 for x: $3(5) = 15$

Substitute 6 for x: $3(\quad) =$

Substitute 7 for x: $3(\quad) =$

Substitute 8 for x:

Substitute 9 for x:

Expression: $x + y$

Substitute 8 for x and 7 for y: $8 + 7 =$

Substitute 3 for x and 5 for y:

Substitute 4 for x and 4 for y:

Substitute 6 for x and ⁻4 for y:

Substitute 9 for x and ⁻9 for y:

Expression:	2x + 3
Substitute 0 for x:	2(0)+3=
Substitute 1 for x:	2( )+3=
Substitute 2 for x:	2( )+3=
Substitute 3 for x:	
Substitute 4 for x:	

Expression:	2x + 3
Substitute ⁻1 for x:	2(⁻1)+3=
Substitute ⁻2 for x:	2( )+3=
Substitute ⁻3 for x:	2( )+3=
Substitute ⁻4 for x:	
Substitute ⁻5 for x:	

Often it's easier to show something in the form of a **table**. Here are some **substitution tables** for you to finish:

x	2x
3	2(3)=6
5	2( )=
8	2( )=
10	
⁻5	

x	x + x
3	3+3=6
5	5+5=
8	
10	
⁻5	

x	−x
⁻5	−(⁻5)=5
5	−(5)=⁻5
⁻8	−(⁻8)=
8	−(8)=
6	−( )=
14	−( )=
0	−( )=

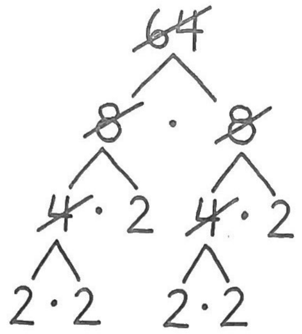
Each table below has *two* variables.

x	y	x + y
3	4	3+4=7
5	2	5+2 =
2	3	
⁻6	3	

x	y	xy
3	4	(3)(4)=
5	2	( )( )=
2	3	
⁻6	3	

Exponents

Remember when we factored the number 64?



$$64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

Here is an easier way to write the answer using an **exponent**:

$$64 = 2^6$$

exponent

base

Do you see what the exponent 6 stands for? It tells how many 2's we have to multiply together to get 64.

Here are some more examples of how we can use exponents:

$5 \cdot 5 = 5^2$ "5 squared"

$5 \cdot 5 \cdot 5 = 5^3$ "5 cubed"

$5 \cdot 5 \cdot 5 \cdot 5 = 5^4$ "5 to the 4th power"

$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^5$ "5 to the 5th power"

Now finish filling in this table:

$6 \cdot 6$	6^2	"6 squared"
$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	3^5	"3 to the 5th power"
$3 \cdot 3 \cdot 3 \cdot 3$		
$7 \cdot 7$		
$2 \cdot 2 \cdot 2$		
$8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$		

Here is another table for you to finish filling in:

	Exponential Form	Factored Form	Multiplied Form
4 squared	4^2	$4 \cdot 4$	16
5 squared			
6 squared			
⁻4 squared	$(^-4)^2$	$^-4 \cdot {}^-4$	
⁻5 squared			
⁻6 squared			
2 cubed	2^3	$2 \cdot 2 \cdot 2$	8
3 cubed			
4 cubed			
⁻2 cubed	$(^-2)^3$	$^-2 \cdot {}^-2 \cdot {}^-2$	
⁻3 cubed			
⁻4 cubed			
2 to the 4th power			
3 to the 4th power			
⁻2 to the 4th power			
⁻3 to the 4th power			
2 to the 5th power			
3 to the 5th power			
⁻2 to the 5th power			
⁻3 to the 5th power			
2 to the 6th power			
3 to the 6th power			
2 to the 7th power			

Use exponents to shorten each expression.

$xxxxx = x^5$

$zzzzzz =$

$aaa =$

$eeeeeee =$

$bb =$

$ssss =$

$xxx =$

$mm =$

$(ab)(ab)(ab) = (ab)^3$

$(^-8a)(^-8a)(^-8a)(^-8a) =$

$(xy)(xy)(xy)(xy) =$

$(4xyz)(4xyz)(4xyz) =$

$(3x)(3x)(3x) =$

$(5a)(5a) =$

$(x+2)(x+2) = (x+2)^2$

$(x+4)(x+4) =$

$(x+2)(x+2)(x+2) =$

$(a-b)(a-b)(a-b)(a-b) =$

Write out each expression the long way.

$x^4 = xxxx$

$c^5 =$

$a^7 =$

$y^3 =$

$b^2 =$

$x^2 =$

$(4x)^3 =$

$(x+5)^2 =$

$(4x)^4 =$

$(x-3)^4 =$

$(xy)^2 =$

$(a+b)^3 =$

Use exponents to shorten each expression .

$aabbb = a^2b^3$

$5xyyyy = 5xy^4$

$6xyyxyxy = 6x^3y^4$

$xxxxyy =$

$10aaabbbb =$

$4abaaba =$

$mmmnn =$

$^{-}3aabb =$

$xyxzxxz =$

$sttttt =$

$6xxyyzz =$

$uvuvuv =$

$uuvv =$

$12mnnn =$

$^{-}8rstsr =$

Write out each expression the long way.

$6x^5y^2 = 6xxxxxyy$

$(ab)^3 = (ab)(ab)(ab)$

$8a^3b^2 =$

$(xy)^4 =$

$x^2y^5 =$

$x^4y^4 =$

$12x^4y =$

$(2x)^3(5y)^2 =$

$x^3y^2z^3 =$

$2x^4 =$

$a^3b^2c =$

$(2x)^4 =$

Here are the answers to the last two problems:

$$2x^4 = 2xxxx$$

$$(2x)^4 = (2x)(2x)(2x)(2x)$$

Did you get them right? Do you see why the answers have to be different? If you pay attention to parentheses you shouldn't have any trouble writing these out the long way:

$5a^4 =$

$(7x)^2 =$

$(5a)^4 =$

$7x^2 =$

$6ab^3 =$

$(xyz)^2 =$

$6(ab)^3 =$

$x(yz)^2 =$

$(6ab)^3 =$

$xyz^2 =$

Multiplying Terms

[illegible] each substitution table.

x	x^2
1	$1^2 = 1 \cdot 1 = 1$
2	$2^2 = 2 \cdot 2 = 4$
3	$3^2 = 3 \cdot 3 =$
4	
5	
6	
7	
8	
9	
10	
11	

x	x^2
$^-1$	$(^-1)^2 = (^-1)(^-1) = 1$
$^-2$	$(^-2)^2 = (^-2)(^-2) = 4$
$^-3$	$(^-3)^2 = (\quad)(\quad) =$
$^-4$	
$^-5$	
$^-6$	
$^-7$	
$^-8$	
$^-9$	
$^-10$	
$^-11$	

x	x^3
1	$1^3 = 1 \cdot 1 \cdot 1 =$
2	$2^3 = 2 \cdot 2 \cdot 2 =$
3	
4	
5	
6	

x	x^3
$^-1$	$(^-1)^3 = (^-1)(^-1)(^-1) =$
$^-2$	$(^-2)^3 = (\quad)(\quad)(\quad) =$
$^-3$	
$^-4$	
$^-5$	
$^-6$	

Equivalent Expressions

Here are two expressions:

$$(3x)(2x) \qquad\qquad 6x^2$$

Let's see what happens when we substitute several numbers for x.

x	$(3x)(2x)$
4	$(3\cdot4)(2\cdot4)=12\cdot8=96$
10	$(3\cdot10)(2\cdot10)=30\cdot20=600$
$^-2$	$(3\cdot{}^-2)(2\cdot{}^-2)={}^-6\cdot{}^-4=24$
5	
3	

x	$6x^2$
4	$6(4)^2=6\cdot16=96$
10	$6(10)^2=6\cdot100=600$
$^-2$	$6(^-2)^2=6\cdot4=24$
5	
3	

You can try substituting some more numbers, but you will find that no matter what number you try, the answers in the two tables always come out the same. We say that $(3x)(2x)$ and $6x^2$ are **equivalent expressions**. This shouldn't surprise you since you already know that multiplication of integers is associative and commutative, so

$$(3x)(2x) = (3\cdot2)(xx) = 6x^2$$

Many times in algebra you will be asked to **simplify** an expression. All this means is that you are supposed to find an equivalent expression that's easier to write.

Simplify each expression below.

$(4x)(5x) = (4\cdot5)(xx) =$

$(3a)(6a) = (\qquad)(\qquad) =$

$(5y)(5y) = (\qquad)(\qquad) =$

$(2x)(7x) = (\qquad)(\qquad) =$

Multiplying Terms

A **term** is a very simple kind of expression where multiplication is the only operation. Here are some examples of terms:

$$5x \quad 3a^2 \quad 8xy \quad x \quad 24a^4bc^3 \quad 7$$

Most terms have two parts — a number part and a variable part. For example, 5 is the number part of $5x$, and x is the variable part of $5x$. The number part is sometimes called the **coefficient.**

When we are multiplying terms, it is easiest to break the problem down into steps. First multiply the number parts of all the terms together. Then multiply the variable parts together.

$(4x)(-5x) = (4 \cdot -5)(x\,x) = -20x^2$

$(4x)(-5x) = -20x^2$

$(7a)(2a) =$

$(-3x)(5x) =$

$(4y)(-3y) =$

$(-6c)(-4c) =$

$(6c)(4c) =$

$(10a)(-2a) =$

$(6x)(5x) =$

$(-7w)(3w)(2w) =$

$(2a)(-5a)(-5a) =$

$(x)(7x)(2x) =$

$(-1v)(4v)(2v) =$

$(2x)(4y) = (2 \cdot 4)(xy) = 8xy$

$(2x)(4y) = 8xy$

$(5a)(2b) =$

$(-4x)(-2y) =$

$(-10a)(4b) =$

$(7u)(-3v) =$

$(-7u)(-3v) =$

$(6u)(-8v) =$

$(6u)(8v) =$

$(5x)(2y)(3z) =$

$(-6a)(b)(7c) =$

$(-3u)(-3v)(-3w) =$

$(2a)(5b)(2c)(3d) =$

$(5t)(-10u)(-3v) =$

Simplify.

$(5x^2)(3x^3) = (5xx)(3xxx) = (5 \cdot 3)(xxxxx) = 15x^5$

$(5x^2)(3x^3) = 15x^5$

$(3x)(5x^3)(4x^2) =$

$(6x^3)(2x^4) =$

$(-4n)(-2n^2)(n) =$

$(5a^2)(-5a^4) =$

$(6a^4)(6a^4) =$

$(-9x)(-4x^3) =$

$(-2a^2)(-2a^2)(-2a^2) =$

$(-9x^2y)(8x^2y^4) = (-9xxy)(8xxyyyy) = -72x^4y^5$

$(-9x^2y)(8x^2y^4) =$

$(2x)(5y)(-6x) =$

$(4a^2b^3)(7a^2b) =$

$(x^2y)(xy)(x) =$

$(-3x^2y^3)(-9x^3y^5) =$

$(-4x)(-8y)(-2x^2) =$

$(9x^2)(6y^2) =$

$(5x^2y)(6x^5y^2) =$

$(3x^4y)(7x^3y) =$

$(-10x^2y^3)(x^4y^2)(9y) =$

$(-5xy)(-9xy)(-2xy) =$

$(4x)(-3y)(2x)(2y) =$

$(5mn)(-9m^3n) =$

$(-6x^5y^3z)(x^4z) =$

$(-4x)(-4x)(-4x) =$

$(x^2y)(x^2y)(x^2y)(x^2y) =$

$(3xy)(3xy)(3xy) =$

$(2xy)(2xy)(2xy)(2xy) =$

$(2x^3)(2x^3)(2x^3)(2x^3) =$

$(4x^4)(4x^4)(4x^4) =$

$(3x^2)(3x^2)(3x^2)(3x^2) =$

$(5x^5)(5x^5)(5x^5) =$

$(2x)(2x)(2x)(2x)(2x)(2x)(2x)(2x)(2x) =$

$(2a)(2b)(2c)(2d)(2e)(2f)(2g)(2h)(2i) =$

Do each problem below in two steps. First write out the problem the long way. Then multiply the terms together.

$(4x)^2 = (4x)(4x) = 16x^2$

$(3x)^2 =$

$(6x)^2 =$

$(7a)^2 =$

$(5w)^2 =$

$(10z)^2 =$

$(3x^2y^3)^2 = (3x^2y^3)(3x^2y^3) = 9x^4y^6$

$(8a^3b)^2 =$

$(10ab^4)^2 =$

$(6xyz)^2 =$

$(a^3b^4)^2 =$

$(-9y^3)^2 =$

$(-4x^2)^3 = (-4x^2)(-4x^2)(-4x^2) = -64x^6$

$(3x^2)^3 =$

$(5a^3)^3 =$

$(-2xy)^3 =$

$(a^2b^5)^3 =$

$(2x^3)^3 =$

$(2x^3)^4 =$

$(2x^3)^5 =$

$(2x^3)^6 =$

$(2x^2)^3(5x)^2 = (2x^2)(2x^2)(2x^2)(5x)(5x) = 200x^8$

$(3x)^4(x^2y)^3 =$

$(3x^4)^2(2x)^3 =$

$(a^2b)^3(ab^3)^2 =$

Finding Powers with a Calculator

With a calculator we can compute a power of a number very easily. To compute 3^5 we could press these keys:

The calculator shows:

[3] [X] [3] [X] [3] [X] [3] [X] [3] [=] → 243

On many calculators we can do this even more quickly. Here's how:

When we enter	[3] [X] [=]	the calculator shows	9	which is 3^2
then press	[=]	the calculator shows	27	which is 3^3
and again press	[=]	the calculator shows	81	which is 3^4
and once more press	[=]	the calculator shows	243	which is 3^5

Notice that we press the [=] key *one less* time than the exponent.

Here's how we would find 2^6:

[2] [X] [=] [=] [=] [=] [=] → 64

Do each problem using a calculator.

[5] [X] [=] [=]

$5^3 =$

$8^4 =$

$2^{10} =$

$7^7 =$

$8^8 =$

[5] [X] [=] [=] [=]

$5^4 =$

$3^6 =$

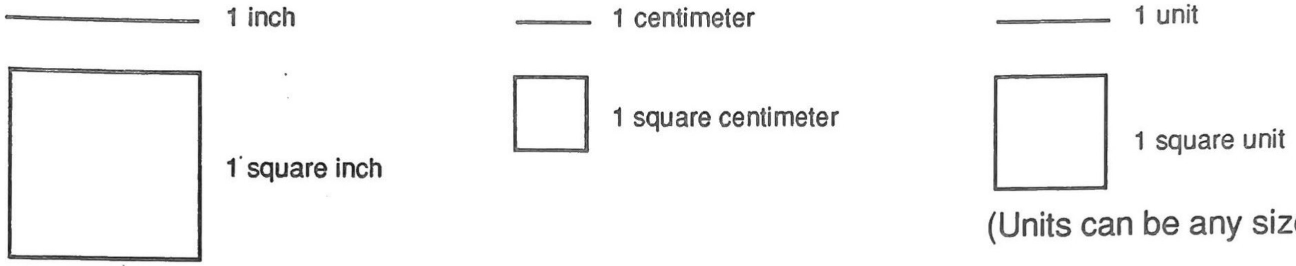
$1^{16} =$

$10^5 =$

$5^{10} =$

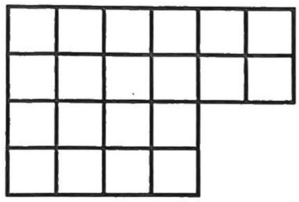
Areas of Rectangles

To measure length we use units like inches or centimeters. To measure area we need a unit which will cover a surface, so we have to use **square units**.

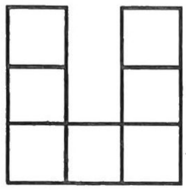


(Units can be any size.)

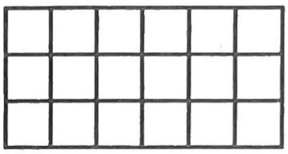
Find the area of each figure by counting the number of square units inside.



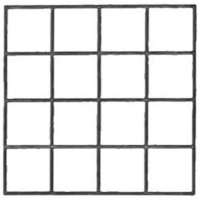
A = ____ sq. units



A = ____ sq. units



A = ____ sq. units

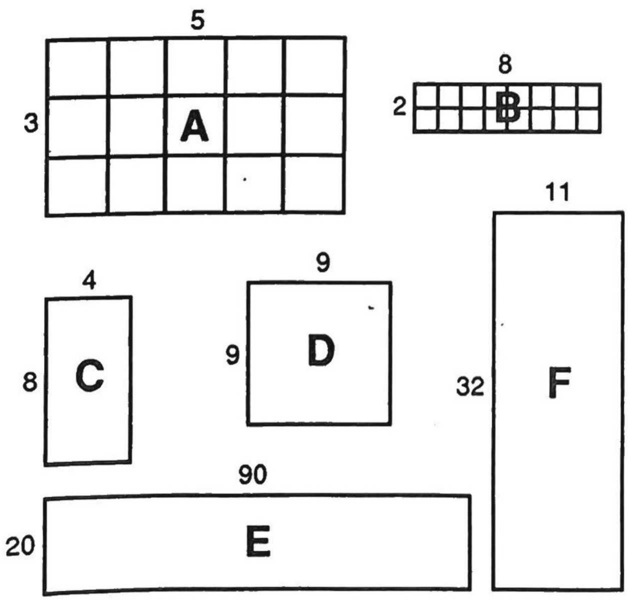


A = ____ sq. units

When the figure is a rectangle we can save time by just multiplying the length by the width to find the number of square units. That's what the **formula** below means.

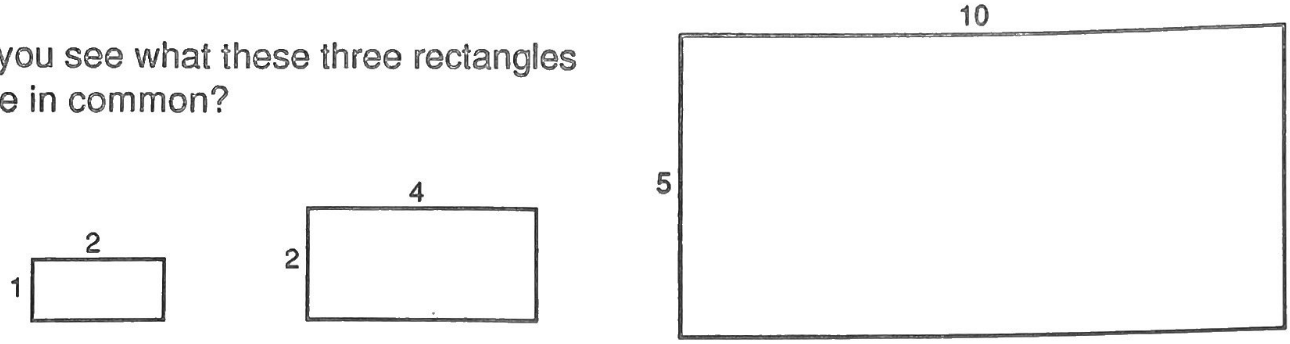
$$A = \ell w$$

Use this formula to find the area of each rectangle below.



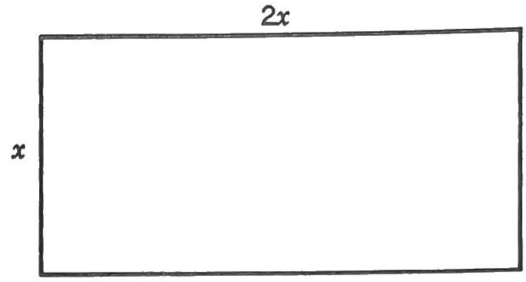
Rectangle	ℓ	w	$A = \ell w$
A	5	3	$A = 5 \cdot 3 = 15$
B			
C			
D			
E			
F			

Do you see what these three rectangles have in common?



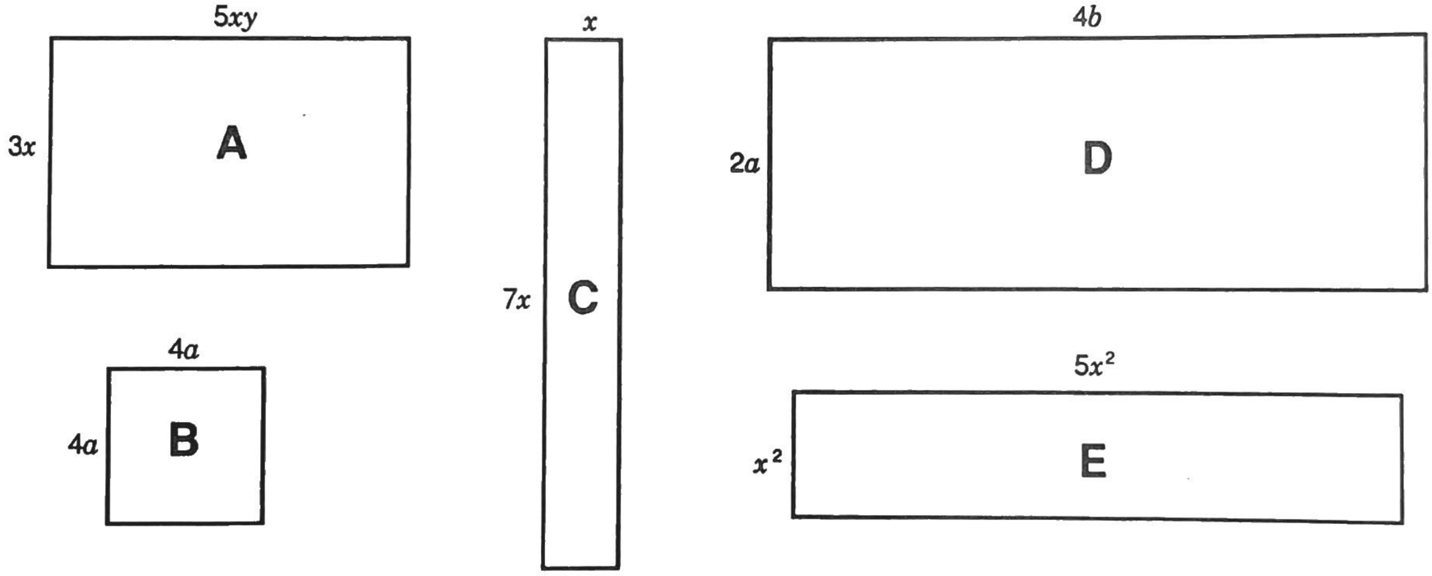
The length of each rectangle is two times its width. We can show a pattern for rectangles of this type by using a variable. If we use x to show the width, then the length is $2x$.

Then we can use the formula $A = lw$ to find an expression for the area.



$$A = lw = (2x)(x) = 2x^2$$

Write an expression for the area of each rectangle below.



Rectangle	l	w	$A = lw$
A	$5xy$	$3x$	$A = (5xy)(3x) = 15x^2y$
B			
C			
D			
E			

Like Terms and Unlike Terms

Terms that have equivalent variable parts are called **like terms**. Terms with variable parts that are not equivalent are called **unlike terms**.

$2x$, $3x$ and $^{-}5x$ are like terms.

$^{-}6a^3$, a^3, $5aaa$ and $32a^3$ are like terms.

8, 1, $^{-}63$ and $^{-}4$ are like terms.

$7x^3y^2$, x^3y^2, $4xxxyy$ and $^{-}6y^2x^3$ are like terms.

Look at each pair of terms and decide if they are like terms or unlike terms. Circle the right answer.

$3x^2$ and $4xx$ (like) unlike	$2a^3$ and $5a^3$ like unlike	$4x$ and $7y$ like unlike
$6x^4$ and $2x^3$ like unlike	$3xy$ and $2yx$ like unlike	$7c$ and 7 like unlike
5 and $^{-}13$ like unlike	$7x^2y$ and $3yxx$ like unlike	$4x$ and $^{-}4x$ like unlike

Match like terms.

$3x$	$6y^2$
$^{-}4y^2$	$7xy$
$17x^4y$	$5x$
$6a^4$	13
$3xxx$	$3x^4y$
$10s$	$9a^4$
$^{-}8$	$^{-}7c$
$6yx$	s
c	$10x^3$

Combining Like Terms

Look at these two expressions:

$$5x + 3x \qquad\qquad 8x$$

When we substitute different numbers for x, here's what we get:

x	5x + 3x
3	5(3) + 3(3) = 15 + 9 = 24
10	5( ) + 3( ) =
$^-2$	5( ) + 3( ) =
5	

x	8x
3	8( ) =
10	8( ) =
$^-2$	8( ) =
5	

As you can see, we get the same answers each time. This happens because $5x + 3x$ and $8x$ are equivalent expressions. We can show that they are equivalent by using the Distributive Principle:

$$5x + 3x = (5 + 3)x = 8x$$

We can always use the Distributive Principle when we are adding like terms. Just add the number parts of the terms; the variable part stays the same. Here is another example:

$$2a^2 + 4a^2 = (2 + 4)a^2 = 6a^2$$

Of course, it's much easier to just write:

$$2a^2 + 4a^2 = 6a^2$$

Here are some expressions for you to simplify:

$10xy + 7xy =$

$5x^2 + {}^-7x^2 =$

$9x^4 + {}^-5x^4 =$

$6a^3 + 4a^3 =$

$^-5s + {}^-3s =$

$4x^2y + 3x^2y =$

Simplify by adding like terms. (Remember, to add like terms you just have to add the number parts. The variable part stays the same.)

$3x^2 + 9x^2 = 12x^2$

$9w + 5w =$

$10a + 4a =$

$^-5yz + 5yz =$

$2y^3 + 8y^3 =$

$3 + {}^-5 =$

$13xy + 13xy =$

$2x^2 + 2x^2 =$

$8a^4 + {}^-2a^4 =$

$5a + 1a =$

$^-7x + {}^-4x =$

$13m^3 + 1m^3 =$

You already know that if x is any integer, then $1 \cdot x = x$. So when you see x in a problem you can change it to $1x$. You can also change a to $1a$, a^2 to $1a^2$, xy to $1xy$, etc.

($4x + x = 4x + 1x = 5x$)

$4x + x =$ **$5x$**

$10a^2 + a^2 =$

$8a + a =$

$x + 3x =$

$2xy + xy =$

$x^2 + 8x^2 =$

$x + 6x =$

$c + c =$

$^-3y + y =$

$^-5x + x =$

$5x + 3x + 4x =$

$6x^2 + {}^-2x^2 + 5x^2 =$

$10ab + {}^-5ab + {}^-3ab =$

$5x + (x + 4x) =$

$^-8n + (3n + 8n) =$

$y^2 + y^2 + y^2 =$

$5x + ({}^-8x + {}^-7x) + 8x + 2x + 7x + {}^-5x =$

$6x^2yz^3 + 3x^2yz^3 + 5x^2yz^3 + {}^-9x^2yz^3 =$

Be careful on these problems! Just add *like* terms.

$4x + 8y + 3x = 7x + 8y$

↖like terms↗

$5y + 8y + 4z =$

↖like↗

$3 + 9b + 10 =$

$8x^2 + 2x^2 + 7x =$

$6xy + 3xy + 3x =$

$^-3ab + ^-10a + ^-8a =$

$7a + 5c + 4c =$

$4x^2 + 9 + 4x^2 =$

$x + 3y + 3x =$

$10x^4 + 8x^4 + 6x^3 =$

$xy + x + xy =$

$a + a + 5 =$

$^-4x^2y + ^-6 + ^-6x^2y =$

$6a + 7b + 5a + 7b = 11a + 14b$

$3x + 6y + 2y + 8x =$

$9x^2 + 10 + 4x^2 + 7 =$

$4x + x + 3x + 8y =$

$7x^2y + 8 + ^-5x^2y + 4 =$

$5a + 3b + 4c + 2a =$

$6x^3 + 9x + 10x^3 + 4x^2 =$

$8a^2 + 4ab + 6a + ^-8a^2 =$

$7a + 5b + c + 4a + ^-3b =$

$(6xy + ^-8xy) + (5xy + ^-2xy) =$

$10x^4 + ^-8x^3 + 4x^3 + ^-5x^2 + 3x =$

$4xy + ^-4xz + 7xy + ^-11xy =$

$8x + 6 + 7x + ^-10 + ^-5x + 8 =$

Here are some expressions with subtraction. Every time you see a subtraction sign, you should add the opposite of the next term. Do all the figuring in your head. Just write down the answer.

($3x + {}^{-}7x = {}^{-}4x$)

$3x - 7x = {}^{-}4x$

($12a + {}^{-}2a = 10a$)

$12a - 2a =$

($5 + {}^{-}8 =$)

$5 - 8 =$

$2xy - 7xy =$

$10x^2 - 6x^2 =$

$6abc - 5abc =$

($5x + 3x + {}^{-}11x =$)

$5x + 3x - 11x =$

$3a^2 - 6a^2 + 10a^2 =$

$10 + 5 - 8 =$

$7xy - 5xy - 5xy =$

$4m + m - 2m =$

$6x^2y - 2x^2y - 10x^2y + 8x^2y =$

$9p - 3p - 9p + 3p =$

$a + 3a + a - 2a + 4a - 2a =$

$9y - 13y =$

$3m - 10m =$

$7ab - 7ab =$

$14a - 9a =$

$3 - 8 =$

$x - 8x =$

$4xy - xy =$

$x^2 - x^2 =$

Simplify.

$9x + {}^{-}5x + x + {}^{-}3x = 2x$

$9x - 5x + x - 3x = 2x$

$5a + 2a - 4a + a =$

$5xy - 2xy - 4xy - 3xy =$

$4x^2 - 9x^2 + x^2 + 2x^2 - 8x^2 =$

$10 - 3 - 2 - 4 =$

$8y - 2y - y - 4y - 5y + 10y =$

$5(ab)^2 + 6(ab)^2 - 4(ab)^2 + 3(ab)^2 =$

$5x - 8x + 3x - 7x + 6x - 4x =$

$8x^4 + x^4 - 6x^4 - 4x^4 - 5x^4 =$

$5ab - 3ab + (9ab - 6ab) + 7ab =$

$7c - 10c + 8c - c - c =$

$4rt + (rt + rt) - 3rt + 5rt =$

$6z - 4z - z + 10z + 3z =$

$x + x + x - x - x + x =$

$5a^2bc^3 - 7a^2bc^3 + a^2bc^3 + 2a^2bc^3 =$

$6 + 5 - 8 + 3 - 10 + 4 - 5 =$

$4m - 7m + 13m - 4m + 7m - 13m =$

$5y + y - 6y - y - y - y =$

Remember, just combine *like* terms.

$3a + 5b + {}^{-}7b = 3a + {}^{-}2b$

$3a + 5b - 7b = 3a - 2b$

$8s - 3s + 4k =$

$10x + 6y - 5x =$

$10x - 6y + 5x =$

$16a + 9 - 7a =$

$a + 2b - 8a =$

$2x - 6y + 7x + 2y =$

$6s^2 - 3s^2 + 4t - 6s^2 =$

$2b + 4 + 3b + 9 =$

$10 - 14xy + 12xy + 21 =$

$3x - 7y + 5x - y =$

$6c - 5 - 2c - 7 - 8d =$

$2x + 5y + 3 + 7x + 2y + 7 =$

$12 + 6p + 3q - 5p + 7q - 2 =$

$a^2 + 5a - 3 - 7a + 6a^2 - 4 =$

$6x^2y + 3xy^2 - 4x^2y - 3xy^2 + x^2y =$

$5 + x - 3 + 2x - x + 7 - 8 + x =$

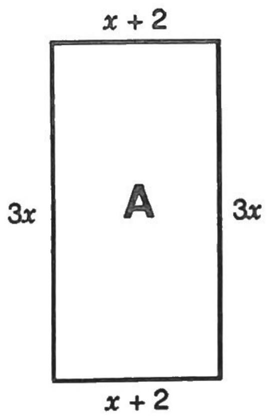
$x^2 - 7x + 4 + x^2 + 4x + 6 =$

$3x^2 + 5 + x^2 - 9 + x^2 + 16 =$

$5a - 4b + 2c - 3b - 6c + a =$

Perimeters

The **perimeter** of a figure is the distance all the way around the outside of the figure. Write an expression for the perimeter of each figure below.



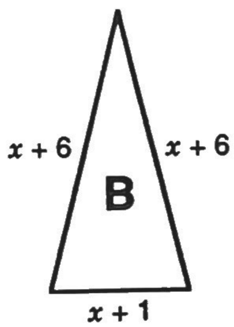
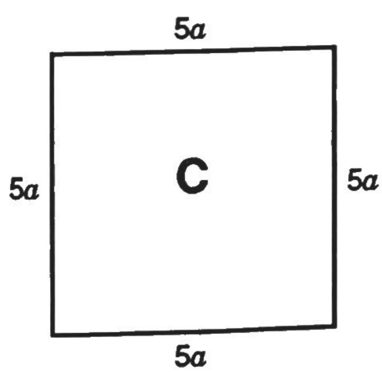
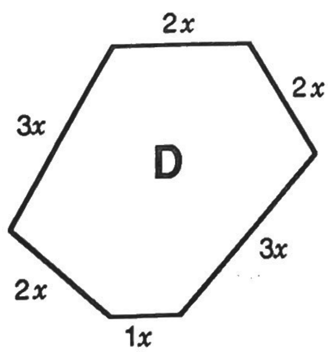
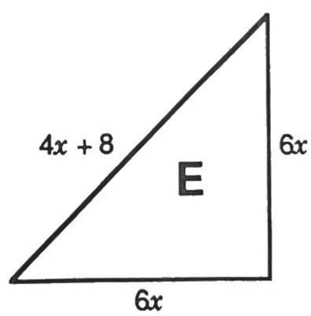
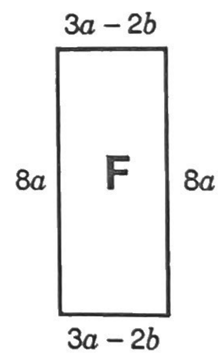


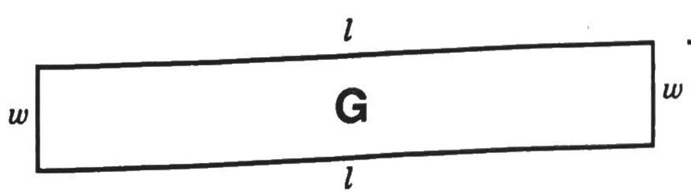
Figure	Perimeter
A	$P = 3x + x + 2 + 3x + x + 2 = 8x + 4$
B	$P =$
C	$P =$
D	$P =$
E	
F	
G	
H	
I	
J	

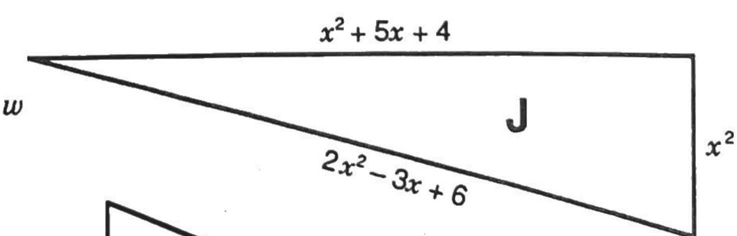


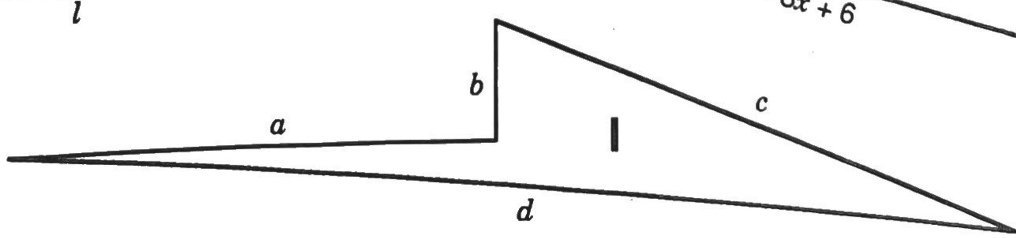


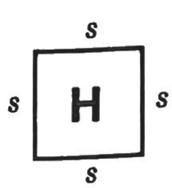






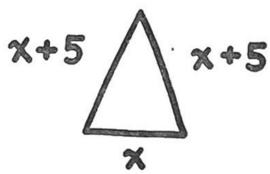






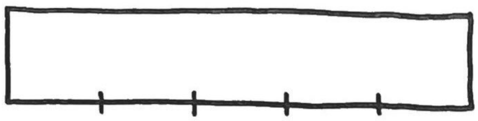
Draw the figure described in each problem. Choose a variable for one side. Then write an expression for each other side and an expression for the perimeter. Simplify the expression for the perimeter if you can.

A triangle with two sides each 5 cm longer than the third side.



$P = x + x + 5 + x + 5 = 3x + 10$

A rectangular dog pen which is 5 times as long as it is wide.



The same dog pen after it is enlarged by adding 3 feet to the length.

A helicopter pad which is a square.

A rectangular rug with a width that is 7 feet shorter than its length.

Another rug which is twice as long as it is wide.

Order of Operations

Up to now the simplifying problems you have done were only one step long. You just had to look at the problem, think, and write down the answer.

In this section are some simplifying problems that take more than one step — so you will have to figure out what to do first. You can tell by using the following rule:

1. If there are *parentheses*, first do what is in them.
2. Then do all the *multiplying*, from left to right.
3. Finally, do the rest of the *adding and subtracting*, from left to right.

Simplify.

$5(4a + 2a)$ $5(6a)$ $30a$	$10y \cdot 2y - 5y \cdot 3y$ $20y^2 - 15y^2$ $5y^2$	$10x - (9x - 2x)$ $10x - 7x$ $3x$
$7(9x - 3x)$	$5(3x^2) - (2x)(4x)$	$14n + (3n - 5n)$
$(6y + 2y)3$	$^-7(4a) - 6a(5)$	$3a - (7a - 2a)$
$4x(3x - 5x)$	$3x \cdot 8y + 2x \cdot 2y$	$(7x^2 - 5x^2) - x^2$
$x^2(4x + x)$	$4a^2 \cdot a - 7a \cdot a^2$	$(11b - 12b) - 5b$

Simplify.

$2x(3x^2+6x^2)$ $2x(9x^2)$ $18x^3$	$(3x^2)(4x)-(5x)(2x^2)$ $12x^3-10x^3$ $2x^3$	$5xy-(4xy-xy)$ $5xy-3xy$ $2xy$
$5a^3(8a-6a)$	$8xy+(3x)(4y)$	$8x^3+(5x^3-3x^3)$
$(4x^3+5x^3)(4x)$	$(5x^2)(4x^3)-6x^5$	$7a-(6a-a)$
$(5x-7x)(6y)$	$(3a)(7a^4)+(4a^3)(3a^2)$	$(4d^2+10d^2)-6d^2$
$(3x^2+x^2)(5y^2-2y^2)$	$(5x^2)(2y^2)-(3xy)(6xy)$	$(5xy-8xy)-4xy$

Using the Distributive Principle

The terms x and 3 are not like terms, so we cannot simplify $5(x + 3)$ by adding the terms in the parentheses. Instead, we use the Distributive Principle.

$$5(x + 3) = 5x + 5 \cdot 3 = 5x + 15$$

$$5(x - 3) = 5x - 5 \cdot 3 = 5x - 15$$

Write an equivalent expression using the Distributive Principle.

$2(x + 6) = \mathbf{2x + 12}$	$2(x - 6) =$	$3(2x + 4) =$
$8(x + 2) =$	$8(x - 2) =$	$11(5x + 2) =$
$6(x + 4) =$	$6(x - 4) =$	$^{-}2(3x + 1) =$
$(x + 3)4 =$	$(x - 3)4 =$	$6(2x - 3) =$
$(x + 9)7 =$	$(x - 9)7 =$	$5(5x - 2) =$
$^{-}3(x + 1) =$	$(x + 1)(^{-}3) =$	$(3x - 10)(^{-}5) =$
$5(x^2 + 6) =$	$(x^2 - 6)5 =$	$(2x^2 + 1)(^{-}3) =$

Simplify.

$8 + 3(x + 2)$ $\mathbf{8 + 3x + 6}$ $\mathbf{3x + 14}$	$x + 4(x - 6)$	$5(2x - 3) + 14$
$^{-}2(x + 7) + 12x$	$x + 3(x - 4) + 2x$	$5x^2 + 3(x^2 - 1)$
$10a + 2(a + 9) + 25$	$5y + (x - 4)(-7)$	$x + 2(x + 1) + x^2$

Evaluating Expressions

Find the number you get for each expression when you substitute 4 for x.

$x + 5$ $4 + 5$ 9	$x + 3$	$x + 10$	$x - 2$	$x - 6$	$x - 4$
$3x$ $3(4)$ 12	$5x$	$9x$	$-3x$	$-5x$	$1x$
$3x + 5$ $3(4) + 5$ $12 + 5$ 17	$2x + 3$	$8x + 4$	$3x - 2$	$5x - 10$	$2x - 10$
$4(x+2)$ $4(4+2)$ $4(6)$ 24	$5(x+3)$	$3(x+1)$	$7(x-1)$	$5(x-2)$	$3(x-7)$
$x(x-2)$ $4(4-2)$ $4(2)$ 8	$x(x+5)$	$x(x-7)$	$x^2 - 2x$ $4^2 - 2(4)$ $16 - 8$ 8	$x^2 + 5x$	$x^2 - 7x$
$-x + 2$ $-(4) + 2$ -2	$-x + 3$	$-x + 4$	$-x + 5$	$-x - 5$	$-x - 4$

Find the value of each expression when $a = 5$, $b = 3$ and $c = 2$.

$a + b$ $5 + 3$ 8	$a + c$	$b + c$	$a - b$	$b - a$	$a - c$
ab $5 \cdot 3$ 15	ac	bc	a^2	b^2	c^2

$a + b + c$	$a - b - c$	$a - (b - c)$	$a - (b + c)$
$a(b + c)$	$ab + ac$	$b(a + c)$	$ba + bc$
abc	$a^2 + b^2$	$a^2 - c^2$	$a^2 c^2$
$(a + b)(a + b)$	$a^2 + 2ab + b^2$	$(a + b)(a - b)$	$a^2 - b^2$

· Find the value of each expression when $x = {}^-3$. Be careful about the signs.

$5x + 15$ $5(^-3) + 15$ $^-15 + 15$ 0	$3x + 4$	$x - 10$	$12 + 2x$	$9 - x$
$2x^2 - 4$ $2(^-3)(^-3) - 4$ $2(9) - 4$ $18 - 4$ 14	$x^2 + 7$	$3x^2 - 2$	$5x^2 + x$	$x^2 - 9$
$x^2 - 2x + 6$ $(^-3)(^-3) - 2(^-3) + 6$ $9 - {}^-6 + 6$ $9 + 6 + 6$ 21	$x^2 + x - 8$	$x^2 + 3x + 1$	$x^2 - 5x - 2$	$2x^2 + 2x - 3$

Evaluate each expression for $a = 5$ and $b = {}^-2$.

$b - 4a$ $^-2 - 4 \cdot 5$ $^-2 - 20$ $^-22$	$a + 3b$	$5a - 6b$	$a + b - 1$	$a - 2b + 1$
ab^2	$(ab)^2$	a^2b	a^2b^2	$a^2 + b^2$
$(a+6)(b-7)$	$(a+3)(b+2)$	$(a-4)(b+5)$	$(a+6)(b-8)$	$(a+4)(b+4)$

Many times we can make it easier to find the value of an expression by simplifying it before we substitute. Look at this example:

Find the value of $x^2 + 3x - 5x + 2x^2 - 9$ when $x = 4$.

Substituting *without* simplifying:

$x^2 + 3x - 5x + 2x^2 - 9$

$4 \cdot 4 + 3 \cdot 4 - 5 \cdot 4 + 2 \cdot 4 \cdot 4 - 9$

$16 + 12 - 20 + 32 - 9$

31

Substituting *after* simplifying:

$3x^2 - 2x - 9$

$3 \cdot 4 \cdot 4 - 2 \cdot 4 - 9$

$48 - 8 - 9$

31

Simplify each expression. Then find its value when $x = 3$.

$5x + 4 - 2x^2 + 7x - 9$ $12x - 2x^2 - 5$ $12 \cdot 3 - 2 \cdot 3 \cdot 3 - 5$ $36 - 18 - 5$ 13	$x^2 + 3x - 5 + x^2 - 3x + 11$
$x^2 + 2x^2 + 3x^2 + 4x^2$	$2x^2 - 5 - x^2 + 5$
$4x^2 + 16 - 3x^2 - 9$	$x^2 + 4x - x^2 + 2x - 1$
$10x - (3x)(2x) + x^2(2x)$	$2x(6x - 5x) + x^2(5x - 4x)$

Written Work

Do these problems on some clean paper. Label each page of your work with your name, your class, the date, and the book number. Also number each problem. Keep this written work inside your book, and turn it in with your book when you are finished. Please do a neat job.

1. Make a substitution table for the expression $5x$. Show what you get when you substitute these numbers for x: 1, 2, 3, 4, 5 and 6. (See page 5.)

2. Make another substitution table for $5x$ that shows what you get when you substitute $^-1$, $^-2$, $^-3$, $^-4$, $^-5$ and $^-6$ for x.

3. Make a substitution table for $x + 8$, and substitute 3, 1, 0, $^-1$, $^-3$, $^-7$, $^-8$, $^-9$ and $^-13$ for x.

4. Sandy and Terry got into an argument. Sandy was saying that:

$$xy^3 = (xy)(xy)(xy)$$

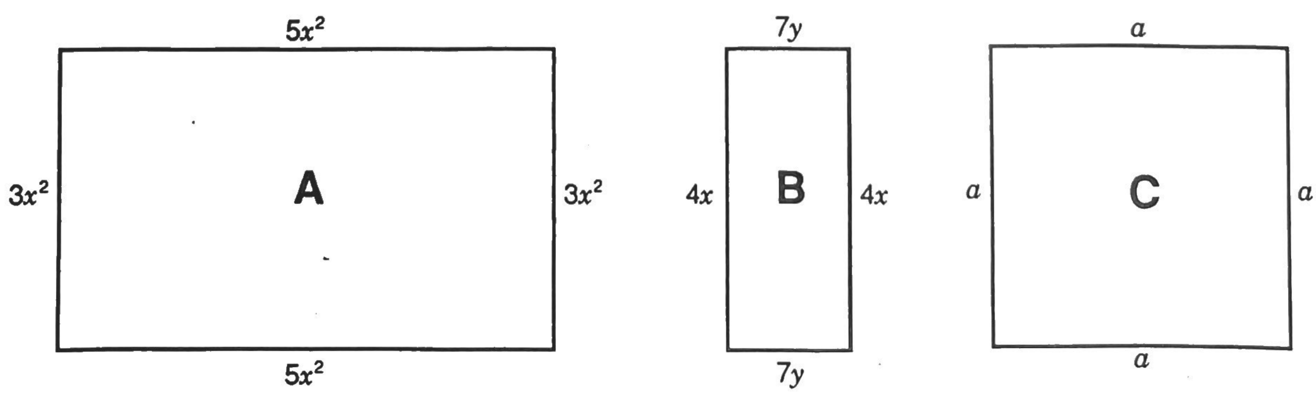
Terry got very upset and said that:

$$xy^3 = xyyy$$

Who was right? Tell why. (See page 9.)

5. Sandy and Terry got into a fight this time. Sandy said that $(3x^2)(3x^5) = 9x^7$, and Terry said that $(3x^2)(3x^5) = 9x^{10}$. Who was right? Explain why. (See page 13.)

6. Write an expression for the area and the perimeter of each figure. Simplify each expression. (See pages 17 and 25.)



7. Copy these problems, leaving plenty of room for your answers. Then simplify each one.

a. Multiplication of Terms (See page 13.)

$(^{-}2xy)(3x)$	$(^{-}5xy^2)(^{-}2y^3)$	$(^{-}3a)(^{-}3a)(^{-}3a)$	$(6x)(^{-}3xy)(^{-}4y)$
$(5r)(^{-}6rs)$	$(^{-}9a^3)(3ab^2)$	$^{-}4a(3ab)(^{-}5b)$	$(^{-}6xy)(8x^2y)$
$(^{-}4z)(z^3)$	$(3ab)(5ab)$	$(^{-}5x)(^{-}6y)(2z)$	$10x^3y(8x^3y)$
$(2x^2)(^{-}3x^3)$	$^{-}7abc(8ab^2)$	$(6n)(^{-}6n)(^{-}6n)$	$(5x)(^{-}3x)(4x)(^{-}2x)$

b. Addition and Subtraction of Terms (See page 24.)

$15 + 4a - 4$	$6m^2 + 10m^2 + 5$	$7p - 2q - 6p - p + 7q$
$7x - 2 + 5x$	$5a - 3 + 4a + 5$	$x + 3y + x + 2y + 5x$
$2y + 8 - 16y$	$3x^2 + 5x - 3x + 2x^2$	$2m - 2n + 3m + 3n$
$2t + 4u - t$	$4xy + 8x - 7xy$	$6 - a + 8 - 2a - 9 + 5a$

c. Mixed

$(^{-}7mn)(4mn)$	$4x - 3y + 5y$	$(4x^2)(3x)(2x^2)(x)$
$^{-}7mn + 4mn$	$(4x)(^{-}3y)(5y)$	$4x^2 + 3x + 2x^2 + x$

8. Do each problem in two steps. First write the problem out the long way, and then multiply the terms together. (See page 14.)

$(3ax)^2$	$(4xy)^2$	$(2x)^3$	$(^{-}2x)^3$	$(^{-}2x^3)^4$	$(3x^2)^4$
$(^{-}5y)^2$	$(x^3y)^2$	$(x^2y)^3$	$(3a^2)^3$	$(x^2y)^4$	$(xy^2)^4$

9. Simplify each expression. (See pages 27 and 29.)

$(2ab + 5ab) \cdot 7a$	$4(5x^2 - 2x^2) + 3x^2$
$(4u)(5v^2) - 2u + uv^2 - 4uv^2$	$4(3x + 5) - 2x - 10$

10. Find the value of each expression when you substitute 2 for a and 7 for b. (See page 31.)

$a(b - 5)$	$ab - 2a$	$a^2 + 2ab + b^2$

Practice Test

Finish each substitution table.

x	$4x$
5	
6	
$^-5$	
$^-6$	
0	

x	$x+6$
5	
6	
$^-5$	
$^-6$	
0	

x	$-x$
5	
6	
$^-5$	
$^-6$	
0	

Use exponents to simplify each expression.

$xxxxxx =$

$3aaaa =$

$^-6xxyyy =$

$2xxxx =$

$(3y)(3y) =$

$(mn)(mn)(mn) =$

$6(ab)(ab) =$

$(2x)(2x)(2x)(2x) =$

Compute.

$5^2 =$

$2^5 =$

$(^-3)^2 =$

$3^3 =$

$8^2 =$

$(^-10)^3 =$

Simplify.

$(8x^3)(2x^2) =$

$(-5a)(3b) =$

$(^-6xy)(^-6xy) =$

$(x^2yz^3)(xy^4) =$

$(^-3xy)(^-2x^2)(-4x^2y) =$

$(3x)(3x)(3x)(3x) =$

$(2a)(3b)(5c) =$

$(xy^2)(xyz)(xyz^3) =$

Simplify.

$5a + 4b - 3a =$

$6x^2 - 8x^2 - 5x =$

$x + 7x + 5 =$

$6ab - 8ab + 4ab =$

$7x - 8y - 3x + 10y =$

$x^2 + 3x + 4 + x^2 + 2x - 6 =$

$a + 7 - 4 + 3a + a - 2a =$

$6a + 4b - 3c - 6b =$

$4x^3 - 2x(10x^2 - 3x^2) =$

$(a + 4a) - 2(6a - 10a) =$

$5x + 7 + 3(x - 7) =$

Do each problem in two steps. First write it out the long way. Then multiply terms.

$(8a)^2 =$

$(3x)^3 =$

$(2x^2)^4 =$

$(x^2y^3)^3 =$

Evaluate each expression for $a = 2$ and $b = {}^-3$.

$10a + 2b$	$a + b$	$a - b$
ab	$3a + 2b$	$a^2 + b^2$