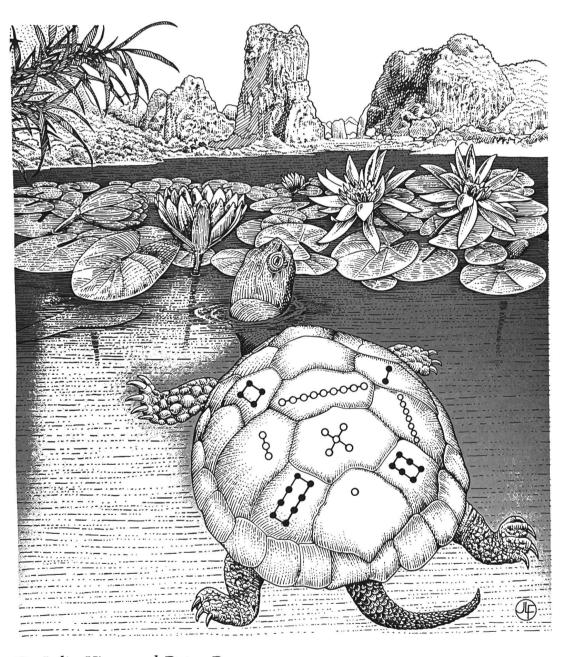
Algebra®



Rational Numbers



Class

By Julie King and Peter Rasmussen

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Systems of Equations

The graph of a linear equation is a line if the equation has two variables and a plane if the equation has three variables. A system of linear equations is a set of such equations considered at the same time.

The history of systems of linear equations had a rather unusual beginning—it started on the back of a turtle. According to ancient Chinese tradition a turtle carried a special square from the river Lo to a man. Here is the square.

Such a square is called a magic square because the three numbers in every row, column and diagonal add up to 15.

The Chinese were especially fond of patterns, so it is not surprising that they would be intrigued by magic squares. About 250 B.C. a book called *Nine Chapters on the Mathematical Art* devoted one entire section to constructing them. It involved three linear equations, marking the first time in history that a system of linear equations was ever encountered.

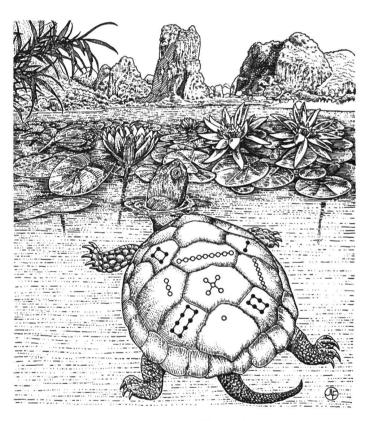
Chinese mathematicians continued to develop and refine techniques for solving systems of linear equations. The peak of this development occurred in 1303 A.D. with the publication of a mathematics book having the unlikely title *Precious Mirror of the Four Elements*. It described a method for solving systems of four equations whose unknowns were called heaven, earth, man and matter.

The results of these Chinese advances remained unknown in the West. During the early part of the 19th century the German mathematician Karl Gauss (1777-1855) introduced an effective method for solving such systems. It was modified slightly by Jordan, and today the procedure is called Gauss-Jordan elimination.

Who is Jordan?

For over a century the name Jordan was assumed to be a tribute to the French mathematician Camille Jordan (1838-1922). However it was discovered in 1986 that the method was actually due to the German geodesist Wilhelm Jordan(1841-1899).

On the cover of this book you see the legendary Chinese turtle emerging from the river Lo with a magic square on its back. The pattern on the turtle's back represents the magic square shown above.



Historical note by David Zitarelli
Illustration by Jay Flom

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Rational Numbers

In Books 1 to 4 we worked with integers (positive and negative whole numbers and 0). We had no trouble adding, subtracting and multiplying integers, but when we got to division we ran into difficulties. Division problems like $9 \div 0$ have no answer, because you can never divide by 0. Other problems, like $10 \div 3$, do not have answers which are integers.

To solve problems like $10 \div 3$ we need a new class of numbers called **rational numbers**. Rational numbers are numbers which can be written as fractions. The **numerator** (top number) and **denominator** (bottom number) of a fraction must be integers and the denominator may not be 0.

$$\frac{3}{4}$$
 $\frac{1}{7}$ $\frac{-5}{2}$ $\frac{0}{8}$ $\frac{-6}{11}$ $\frac{-9}{1}$ $\frac{-7}{3}$ denominators

Every integer is a rational number because it can be written as a fraction with a denominator of 1. Rewrite each integer as a fraction.

$$8 = \frac{8}{1}$$
 $-3 = \frac{-3}{1}$ $0 = -15 = 4 = 25 = 6 = 6$

Every mixed number is a rational number because it can be written as a fraction. Rewrite each mixed number as a fraction.

Every decimal is also a rational number (unless it goes on forever without repeating). A **terminating decimal** (one that comes to an end) is a rational number because it equals a fraction with a denominator of 10 or 100 or 1000, etc. Rewrite each terminating decimal as a fraction.

$$0.6 = \frac{6}{10}$$
 $0.9 = 1.3 = \frac{13}{10} = \frac{13}{10}$
 $0.06 = \frac{6}{100}$ $0.09 = 2.7 = 0.006 = 0.19 = 5.01 = 0.056 = 0.119 = 3.27 =$

Dividing Integers

Now we can divide any integer by any other integer except 0. All we have to do is write a fraction with the dividend (the number we are dividing into) as the numerator (top) and the divisor (the number we are dividing by) as the denominator (bottom).

$$10 \div 3 = \frac{10}{3}$$

Do each division problem. If the divisor goes evenly into the dividend, write your answer as an integer. Otherwise, write it as a fraction.

$$|2 \div ^{-}2 = ^{-}6$$
 $|5 \div 3 =$

$$15 \div 3 =$$

$$-8 \div -3 =$$

$$40 \div 7 = \frac{40}{7}$$
 $-7 \div 2 =$

$$-2 \div 19 =$$

A fraction can be positive or negative. To find the sign, just follow the rules for division. When division is written using a fraction bar, the rules look like this:

If a fraction is positive, we will write it with no signs. If a fraction is negative, we will write it with the negative sign on top.

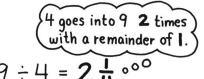
$$\frac{3}{4}$$
 is positive, so we will write $\frac{3}{4}$

$$\frac{-3}{4}$$
 is positive, so we will write $\frac{3}{4}$. $\frac{3}{4}$ is negative, so we will write $\frac{-3}{4}$.

Do each division problem. Write your answer as a positive or negative fraction.

$$13 \div -3 = \frac{-13}{3} -4 \div -7 =$$

Divide. Write your answer as an integer or as a positive or negative mixed number.



7 goes into 12 1 time with a remainder of 5. The answer is negative.

$$-12 \div 7 = -15 \circ$$

$$-18 \div 5 =$$

$$20 \div 3 =$$

$$-14 \div -5 =$$

$$-11 \div 4 =$$

$$-37 \div 10 =$$

Divide. This time write your answer as a positive or negative decimal.

$$|07 \div |00 = \frac{107}{100} = 1\frac{7}{100} = 1.07$$

$$15 \div 4 = 3.75$$

$$-17 \div 2 =$$

$$-12 \div 5 =$$

Equations with Rational Solutions

Now we can use the Division Principle to solve equations even when the answer is not an integer. Solve each equation. Write your answer as a fraction or as a mixed number.

$\frac{7x}{7} = \frac{-25}{7}$ $x = -3\frac{4}{7}$	9x = 40	-4x = 17
2x-5=14	3 _x + 7 = ⁻ 4	-5x +1 = 15
x - 3 = 8x + 5	-2(x -5) = 7	χ - 5χ + 7 = ⁻ 8

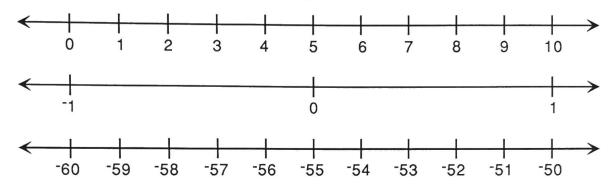
Solve these equations, too. This time if your answer is not an integer, write it as a decimal.

$\frac{4x}{4} = \frac{-10}{4}$ $x = -2.5$	-5x = 18	$\frac{-2x}{5} = 7$
7x - 7 = 3x + 20	x-9=6x+7	$\frac{10x-2}{5}=3$
3(x-5)=x-20	4(x+6) = 23	2(x-3)+x=9

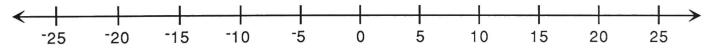
Number Lines

In Book 1 we used **number lines** to help us think about adding and multiplying integers. The football field was a kind of number line. Rulers and the scales on thermometers are also number lines.

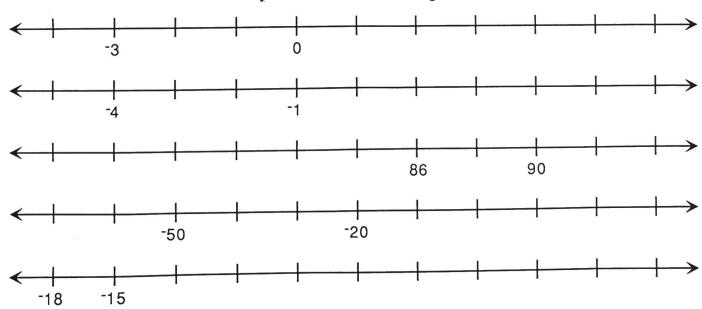
To make a number line we draw a line and divide it into sections of equal length called **units**. Then we number the points which separate the units. Here are three number lines:



The arrows on the ends of each number line show that the number line keeps going. We can start with any number as long as we number the points in order (usually from left to right). Sometimes we do not show every unit. This number line only shows every fifth unit:



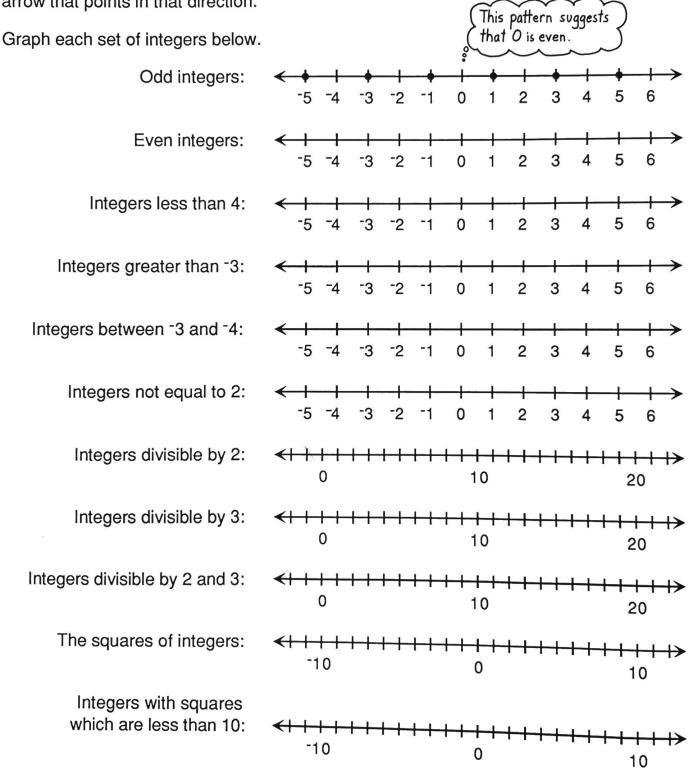
Here are some number lines for you to finish numbering:



Make a number line showing all the integers from ⁻⁵ to 5.

Graphing Integers

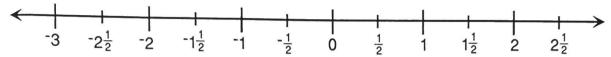
We can use a number line to picture a set of numbers. On the number line we make a dot to show each number in the set. This is called a **graph** of the set. A graph can help you see a pattern or answer a question. If a pattern continues forever to the left or right, we fill in the arrow that points in that direction.



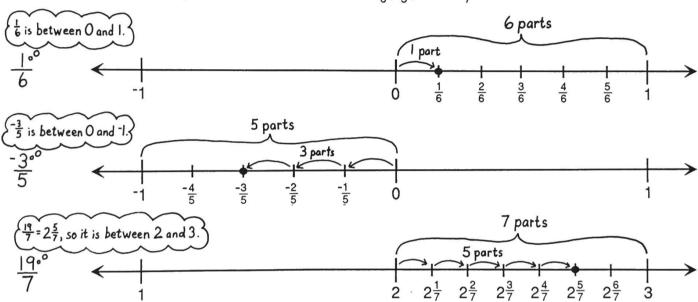
Did you notice any interesting patterns in the graphs you made?

Graphing Rational Numbers

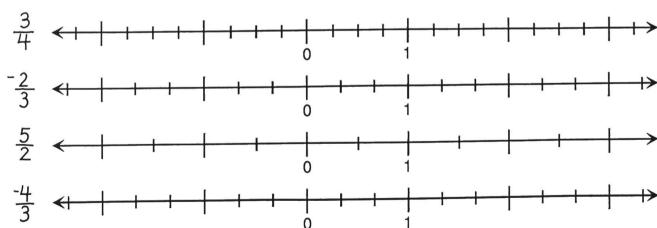
Integers are not the only points on a number line. On the number line below we have also labeled the points halfway between each integer and the next.



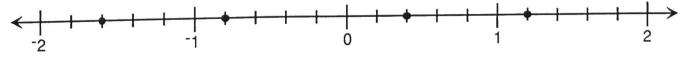
In fact, there is a point on the number line for *each* rational number. To find this point, first write the rational number as a fraction. The denominator of the fraction tells how many parts to divide each unit of the number line into. The numerator tells how many parts to count off to the right of 0 (if the number is positive) or to the left of 0 (if the number is negative) to find the point. Here's how to find $\frac{1}{6}$, $\frac{3}{5}$, and $\frac{19}{7}$.



On each number line, first finish labeling the points. Then graph the rational number at the left.

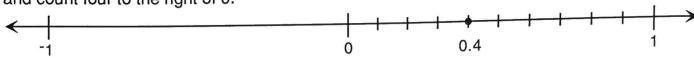


Label each rational number shown on the number line below.

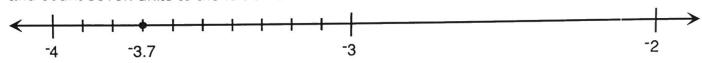


Each decimal is a rational number (unless it goes on forever without repeating), so it also has a place on the number line. To find the point for a decimal, think of it as a fraction or mixed number.

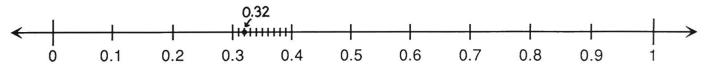
0.4 is the same as $\frac{4}{10}$. This number is between 0 and 1 so we divide that unit into ten parts and count four to the right of 0.



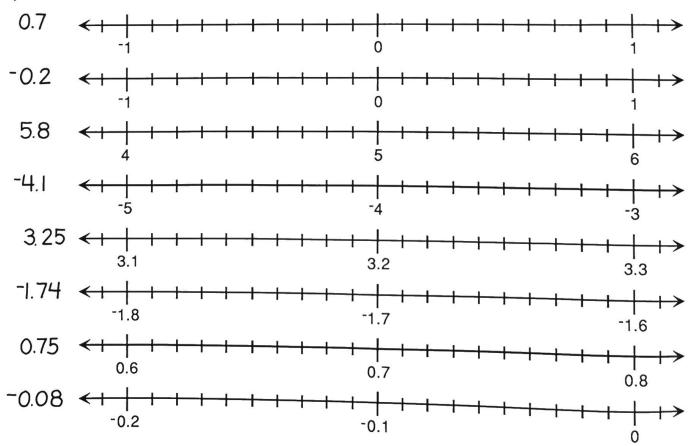
 $^{-3.7}$ is equal to $^{-3}\frac{7}{10}$. This number is between $^{-3}$ and $^{-4}$ so we divide that unit into ten parts and count seven units to the left of $^{-3}$.



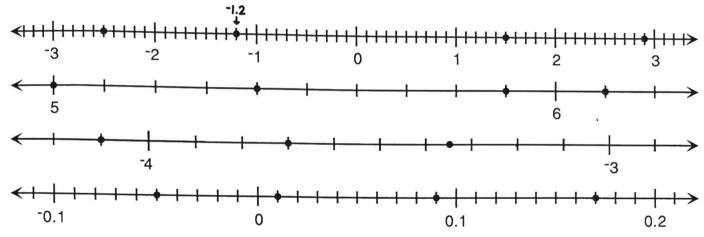
For hundredths we could divide the unit into a hundred parts, but to save time it makes sense to divide it into tenths first and then to divide only one of the tenths into ten parts. To graph 0.32 this way we first notice that it is between 0.3 and 0.4. Then we divide the section between 0.3 and 0.4 into ten parts. Each of these parts is a hundredth of the unit.



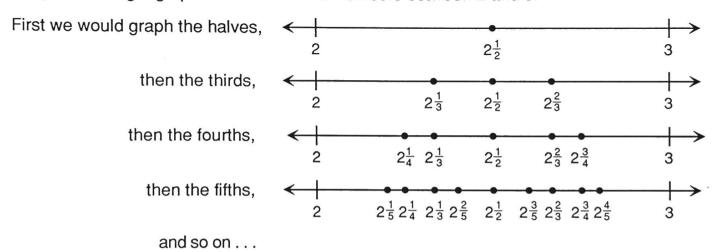
Graph each decimal below.



Find a decimal name for each point graphed on the number lines below.



Imagine making a graph of all the rational numbers between 2 and 3.



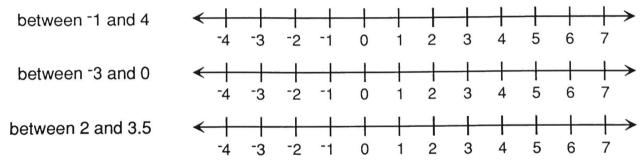
We would never be finished! Soon the line would be so crowded with dots that you couldn't tell one from another. So when we want to show *all* the rational numbers between 2 and 3 we just shade the whole section of the line between those numbers.



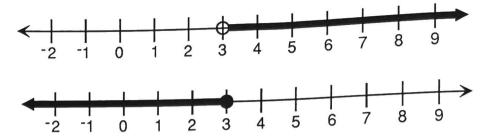
Whenever we say "between" we will mean "not including the endpoints."

We have used hollow dots at 2 and 3 to show that those numbers are not included.

You graph all the rational numbers which are:



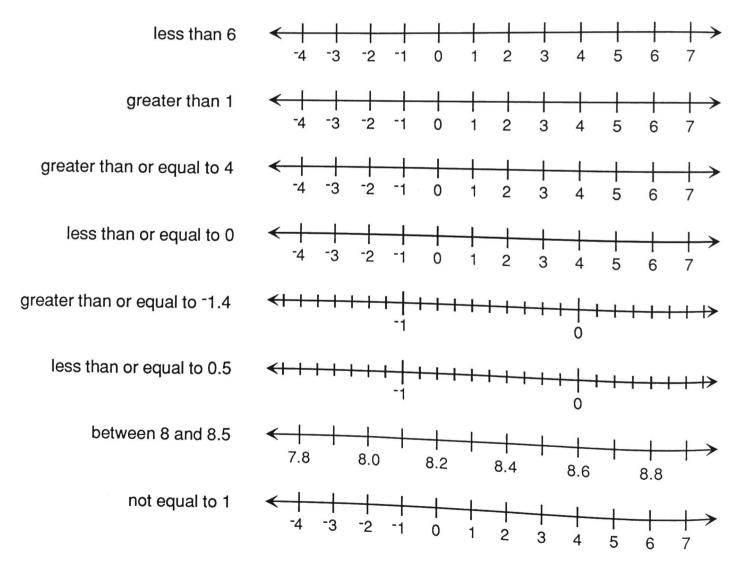
Can you tell what sets have been graphed?



The first graph shows all rational numbers which are greater than. 3. The hollow dot shows that 3 is not included.

The second graph show all rational numbers which are less than or equal to 3. This time 3 *is* included, so we have used a solid dot. On both graphs the arrows have been filled in to show that the graphs continue.

Graph all the rational numbers which are:



Inequalities

In Book 3 we worked with equations. Remember that an equation is a sentence about numbers being equal, like x + 4 = 5.

Another kind of sentence is an inequality — a sentence about numbers being unequal. Here are two examples of inequalities:

$$x + ^{-}4 < 5$$

means

"x + -4 is less than 5."

$$x + -4 > 5$$

means

"x + -4 is greater than 5."

Sometimes we combine two symbols to make a new symbol. The symbol ≤ means "is less than or equal to." And the symbol ≥ means "is greater than or equal to." Sentences using these combined symbols are also inequalities.

$$x + -4 \le 5$$

means

"x + -4 is less than or equal to 5."

$$x + -4 \ge 5$$

means

"x + -4 is greater than or equal to 5."

To get used to using <, >, \le and \ge , read the following statements carefully.

Each of these is true:

Each of these is false:

Mark each statement below true (T) or false (F).

$$18 > 2$$
 T

_		
	٦,	L
1	7	7)
-	-	

is the international road sign for "Passing Permitted."



is the sign for "No Passing."

Can you figure out what each sign below means?

-	
K	(2)
	Y
-	

is the sign for



is the sign for



is the sign for



is the sign for

A slash is used on international road signs to mean "No." We use the same idea to make up new symbols in algebra. In these symbols the slash means "is not."

$$7 \neq 4$$

means

"7 is not equal to 4."

0 > 5

means

"0 is not greater than 5."

2 ≰ 2

means

"2 is not less than 2."

10 1 12

means

"10 is not greater than or equal to 12."

3 ≰ 0

means

"3 is not less than or equal to 0."

You write the meaning of each sentence below.

8 44

means

 $0 \geqslant 3$

means

5 ≱ 6

means

 $-10 \neq 10$

means

 $x + 4 \neq 9$

means

-2× ≮ 10

means

Š ≱ 5

means

A number is a **solution** of an inequality if it makes the inequality true when you try it in place of x.

These numbers are solutions of x + -4 < 5:

7 because $7 + ^{-4} < 5$

 $^{-1}$ because $^{-1} + ^{-4} < 5$

0 because 0 + -4 < 5

These numbers are *not* solutions of $x + ^-4 < 5$:

10 because 10 + -4 < 5

9 because 9+-4 < 5

25 because 25 + ⁻4 ≮ 5

Try to find at least five integers which are solutions for each equation or inequality. If there aren't five integer solutions, list as many as you can find.

Absolute Value

When we multiply or divide integers we get the amount of the answer by multiplying or dividing and the sign of the answer by following the rules for signs. The amount of a number is often called its absolute value. We put the symbol | | around a number when we want to talk about its absolute value.

means

"The absolute value of 6 is 6."

$$| -6 | = 6$$

means

"The absolute value of ⁻6 is 6."

$$|0| = 0$$

means

"The absolute value of 0 is 0."

Finding the absolute value of a number is easy. Just get rid of its sign. You find each absolute value below.

$$\left| \frac{-2}{3} \right| =$$

Use trial and error to find as many solutions as you can for each equation below.

absolute values of 5.

$$|x| = 5$$
 5, -5

$$|x| = 8$$

$$|x| = 0$$

$$|x-5|=2$$

$$|x| = -5$$

For each inequality below, try to find at least five integers which are solutions.

7, 10 and 12 have absolute values greater than 6. but so do -7, -10 and -12.

|x| < 6

 $|x| \ge 0$

|x| # 3

1x1 ≮ 7

1x1 × 10 ____

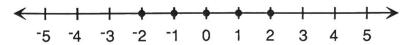
|x| > x

 $|x| \leq 2$

Graphing Inequalities

Look at the last equation on the previous page. The five integers which are solutions are -2, -1, 0, 1 and 2. We could easily make a graph of this set of solutions.



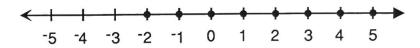


Some inequalities, like x > 3, have an infinite number of solutions. It would be impossible to list all the integers which are solutions, but we could show the solution set by *starting* a list and then using three dots to show that it continues on and on.

$$x > -3$$
 {-2,-1,0,1,2,3,...}

We could also graph the set of solutions using a darkened arrow on the right to show that the dots continue to the right.

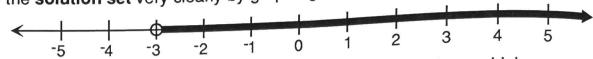
$$x > -3$$
 {-2,-1,0,1,2,3,...}



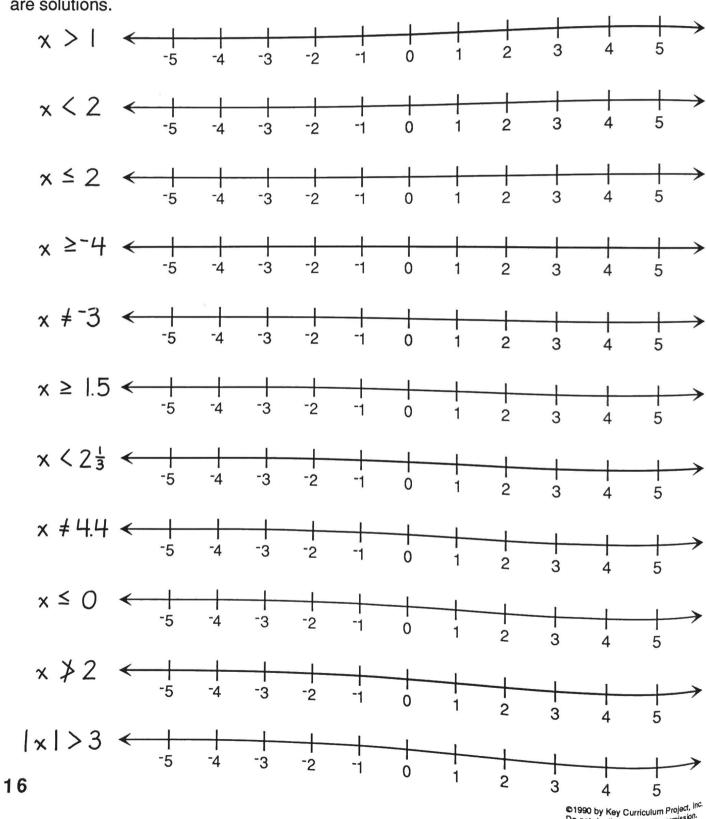
For each inequality, show the integers which are solutions in two ways: by making a list and by graphing.

, 5	1		List		Graph
× <	. 1	{		}	<++++++++++++++++++++++++++++++++++++
x >	>5	{		}	<
χ <	-3	{		}	<
χ ≥	- 4	{		}	<
x >	210	{		}	<
x ≤	0	{		}	<
x	< 4	{		}	<
x	≥ 2	{		}	<
x +	1≥5	{		}	<

It would be impossible to list all the rational numbers which are solutions of the inequality x > 3. 3 is not a solution, but every rational number greater than 3 is. We can show the **solution set** very clearly by graphing.



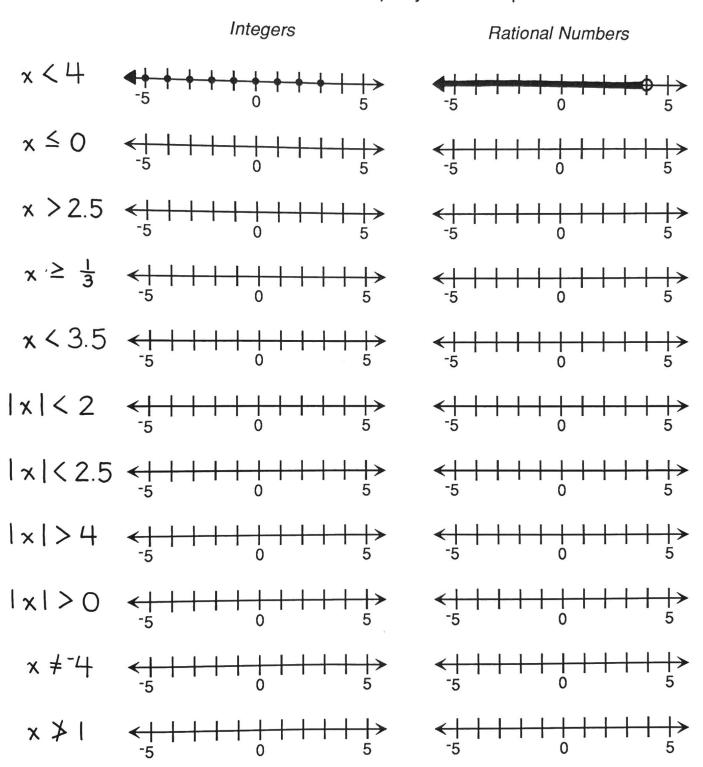
For each equation or inequality below, graph the set of all rational numbers which are solutions.



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The graph of an inequality depends on what kinds of numbers we allow as solutions. The set of numbers we allow as solutions is called the **replacement set**.

In the first problem below we have shown what the graph of x < 4 looks like when only integers are allowed as solutions and what it looks like when all rational numbers are allowed as solutions. Make a graph of each inequality for each replacement set.



From now on in this book we'll always use rational numbers as our replacement set.

Solving Inequalities

We can tell if a number is a solution of an equation or inequality by substituting that number for the variable and seeing whether the result is true or false.

Is 8 a solution of
$$2x+5 < x-15$$
?

We can substitute 8 for x to find out:

$$2x+5 < x-15$$

$$2x+5 < 8-15$$

$$16+5 < 8+15$$

$$21 < -7$$
So 8 is not a solution.

false

Luckily we do not have to substitute every time we want to check a possible solution. We can use the Addition Principle to help us solve an inequality just as we used it to solve equations. The Addition Principle can help us find a simpler inequality with the same solution set. This is called **solving** the inequality. Look at this example:

$$2x^{-x} + 5 < x^{-x} - 15$$

 $x + 5^{-x} < -15^{-5}$
 $x < -20$

Only numbers less than $^{-}20$ are solutions, so 8 could not be a solution. Neither could any other positive number.

Solve each inequality below using the Addition Principle.

x + 19 ⁻¹⁹ ≥ 3 ⁻¹⁹ x ≥ -16	x - 4 < 12	x + 9 ≤ -6
-2 + x < 6	+ 🗙 ≥	x - 5 < -3
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	10 < x + 6	12 ≥ x - 7
$5x^{++}$ > $4x^{++}$ 15 x + 1 > 15 x > 14	7x - 9 < 6x + 11	$x-5 \leq 2x+8$

Solve each inequality below using the Addition Principle. Draw a graph of each solution set.

Solve each friequality below using the Addition Principle. Draw a graph of each solution set.			
x-7 >0 x >7	x - 9 < -9	6 ≤ x + 12	
← 1	←		
2x + 1 > x - 4	x + 16 < 2x + 20	5x - 6 ≤ 4x + 1	
	← →	←	
3(x-3)>2x	x - 4 + 11 ≤ 26	2(x-1) < 3(x+4)	
	←	←	
$x^2 - 5 < x^2 + x + 1$ -5 < x + 1	$x^2 + x + 8 \ge x^2 - 4$	$2x^2 + 22 > 2x^2 + x - 9$	
-6 < x x > -6			
-7 -6 -5 -4 -3 -2	← →	← →	
$(x+2)(x-2) < x^2 + x$	$(\chi + 2)(\chi - 1) \geq \chi^2$	$(x-4)(x+5) < x^2-19$	
x²-4 < x²+x -4 < x			
x > -4			
\leftarrow	←	←	

Maybe you are wondering whether there are Multiplication and Division Principles for Inequalities. The answer is yes, but they aren't quite the same as the Multiplication and Division Principles for Equations. When we multiply or divide both sides of an inequality by a *positive* number, we do get an equivalent inequality. But when we multiply or divide both sides by a *negative* number, we must *reverse the inequality sign* to get an equivalent inequality. Look at these sentences to see why:

Multiplying by 2:	$^{-6}$ < 10 \leftarrow true $^{-6^{\circ 2}}$ < 10° 2 \leftarrow true $^{-12}$ < 20	Dividing by 2:	-6 < 10 ← true -6 < ½ ← true -3 < 5
Multiplying by ⁻ 2:	$\begin{array}{c} -6 < 10 \leftarrow \text{true} \\ -6^{-2} < 10^{-2} \leftarrow \text{false} \\ 12 < -20 \leftarrow \text{true with } < \\ 12 > -20 \leftarrow \text{switched to} > \end{array}$	Dividing by ⁻ 2:	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$

Solve each inequality using the Multiplication and Division Principles. Remember to switch the direction of the inequality sign if you multiply or divide by a negative number.

The direction of the inequality s (5 is positive. Leave the \geq sign alone.) (6) $\frac{5x}{5} \geq \frac{-30}{5}$ (7) $\frac{5x}{5} \geq \frac{-30}{5}$ (8) $\frac{5x}{5} \geq \frac{-30}{5}$	Ighth you multiply of divide by a constraint of the constraint of	
$\frac{7-6}{-9x} \le 72$	x > -24 - x < 8	$\frac{x}{3} \geq -4$
<u>-</u> × ≥ -3	-x=1x, $-x > 4$	11x < -33
$-\frac{x}{5} \le 12$	2x > -5	-3x < 0

Maybe you are wondering whether there are Multiplication and Division Principles for Inequalities. The answer is yes, but they aren't quite the same as the Multiplication and Division Principles for Equations. When we multiply or divide both sides of an inequality by a *positive* number, we do get an equivalent inequality. But when we multiply or divide both sides by a *negative* number, we must *reverse the inequality sign* to get an equivalent inequality. Look at these sentences to see why:

Multiplying by 2:	$^{-6}$ < 10 \leftarrow true $^{-6^{\circ}2}$ < 10° 2 \leftarrow true	Dividing by 2:	0	← true ← true
	-12 < 20 <		-3 < 5	
	-6 < 10 ← true	Dividing	0 1.0	← true
Multiplying by -2:	$-6^{-2} < 10^{-2} \leftarrow \text{false}$	Dividing by ⁻ 2:	<u>-6</u> < <u>-10</u> ≥	← false
	12 <-20		3 < ⁻ 5	true with <
	12 > -20 ← true with < switched to >		3 > -5	switched to >

Solve each inequality using the Multiplication and Division Principles. Remember to switch the direction of the inequality sign if you multiply or divide by a negative number.

5 is positive. Leave the \geq sign alone. $5 \times 5 \times 2 = 30$ $5 \times 5 \times 2 = 30$ $5 \times 2 \times 30$ $5 \times 2 \times 30$ $5 \times 2 \times 30$	Switch < to >. $\frac{x}{-4}$ < 6 $\frac{x}{-4}$ < 6 $\frac{x}{-4}$ > (6). 4 $x > -24$	x < 7
-9 _x ≤ 72	<u>*</u> < 8	x / 3 ≥ -4
- % ≥ -3	$-x=1x$, \rightarrow 4	IIx < ⁻33
$\frac{x}{-5} \leq 12$	2x > -5	-3x < 0

Solving each inequality below takes more than one step. Remember to switch the inequality sign whenever you multiply or divide both sides by a negative number.

inequality sight whenever you multiply or divide both sides by a negative number.			
$ \begin{array}{c} $	5x +1 ≤ -44	-2x + 15 < 7	
$\frac{\times + 15}{2} < -5$	$\frac{x+9}{-3} > 3$	$\frac{x-6}{-10} \geq -4$	
- <u>5x</u> ≥ 10	$\frac{3x}{7} < 1$	$-\frac{4x}{3} > -12$	
× + 14 > 9	x + 5 ≤ 12	$\frac{x}{2} - 10 < -19$	
2x - 7x ≤ 3	-3(χ-5) > 21	9 - x ≤ 7	

Here's how Sandy and Terry solved the last inequality on page 21.

Sandy
$$7 - x^{+x} \le 7^{+x}$$

$$9^{-7} \le 7^{+x}$$

$$-x \le -2$$

$$1x \le -2$$

$$x \ge 2$$

$$x \ge 2$$

$$7 \times x \ge 2$$

$$7 \times x \ge 2$$

Both Sandy and Terry ended up with the same solution set. Whose method do you like better? Why?

Both methods work. Use either one to solve each inequality below.

Both methods work. Ose either one to solve each mequality bolow.			
2-x > 16	25 ≤ 10 - x	-5x + 9 ≥ 3x	
3x + 15 > 3 + 7x	x + 20 < 5x - 8	18 + x ≥ 12 + 7x	
5(3-α) < 50	$\frac{4-x}{2} > 12$	12 - 7x ≤ 12	

Solve each inequality and graph the solution set.

$$3x - 4x > 6$$

$$x + 12 \geq 2x - 5$$

$$\leftarrow$$

$$\frac{x-7}{2} < -6$$

$$\frac{x}{2}$$
 - 7 < -6

$$7x - 2x - x \ge 24 + 3x$$

$$\frac{4x}{3} + 1 < 4$$

$$\longleftarrow$$

$$-16 + 4x > 10 - x$$

$$3(x-4)-9x \ge 2x-4$$

When an absolute value sign appears in an equation or inequality, you should *not* use the Addition, Multiplication or Division Principle to simplify an expression *inside* the absolute value sign. Instead, think about what the absolute value sign means.

The number inside the I I sign could be either 6 or 6.

$$|x + 1| = 6$$

$$x + | = 6 \text{ or } x + | = -6$$

$$x = 5 \text{ or } x = -7$$

To make sure that both 5 and $^{-}$ 7 are solutions, we can substitute each for x.

Check:
$$|5+1| = |6| = 6$$

 $|-7+1| = |-6| = 6$

Solve each equation, and check your solutions.

$$|x+5|=7$$

$$|x-10|=2$$

Check: Check:

$$\left|\frac{x}{5}\right| = 2$$

Check: Check:

$$|3x + 4| = 10$$
 $|12x - 6| = 6$

Check: Check:

Solve each equation.

The 3 is outside the 1 I sign, so I can use the Addition Principle.

$$|x| + 3^{3} = 20^{-3}$$

$$|x| = 17$$

$$x = 17 \text{ or } -17$$

$$|x| - 8 = -2$$

$$|x| + 8 = 11$$

$$2|x|-5=3$$

$$|4x-5|=7$$

$$4|x|-5=7$$

$$|x+2|-3=7$$

$$|x+5|+4=12$$

$$|2x-3|+2=|1$$

$$-3|x|+5=-4$$

Solve and check each inequality.

You can't check all the solutions, so pick a few samples.

x +2 <
Numbers from 9 x < 9
Check: $4 < 9$ and $4 > -9$ 6: $ 6 + 2 = 6 + 2 = 8 < 11$
6: 6 + 2 = 6 + 2 = 8 < 11
-4: 1-41 + 2 = 4 + 2 = 6 < 11
x > 12

amples. |
$$x - 2 | > 3$$

 $x - 2 > 3$ or $x - 2 < -3$
 $x > 5$ or $x < -1$
Check: $| 9 - 2 | = 7 > 3$
 $| 4 - 2 | = | -6 | = 6 > 3$
 $| x | \le 4$

Check:

Check:

$$|x| + 4 < 20$$

|x|-6 > 1

Check:

$$|x-1| < 5$$

|x + 10| > 5

Check:

Check:

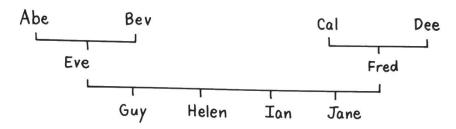
$$|12x| \geq 24$$

13/58

Check:

Relations

In algebra, =, <, >, \le and \ge are sometimes called **relations**. Thinking of relations in your family can help you understand relations in algebra. Relations in your family are people connected to you in certain ways. You have different kinds of relations: mother, father, brothers, sisters, aunts, uncles, etc. Look at this family tree:



The tree shows Abe is married to Bev, and Eve is their child. Cal is married to Dee, and their child is Fred. Guy, Helen, Ian and Jane are the children of Eve and Fred.

Let's see how some of the people in this family are related:

Sisters: We will call the sister relation S. S(x) means "a sister of x."

S(Guy) = Helen	means	"A sister of Guy is Helen."
S(Guy) = Jane	means	"A sister of Guy is Jane."
S(lan) =	means	
S(lan) =	means	
S(Jane) =	means	
S(Helen) =	means	

Brothers: We will call this relation B. B(x) means "a brother of x."

$$B(lan) =$$

$$B(Jane) =$$

$$B(Jane) =$$

$$B(Guy) =$$

$$B(Helen) =$$

$$B(Helen) =$$

Fathers: We will call this relation F. F(x) means "the father of x."

$$F(Eve) =$$

$$F(lan) =$$

$$F(Guy) =$$

$$F(Fred) =$$

$$F(Helen) =$$

$$F(Jane) =$$

Grandmothers: We will call this relation G. G(x) means "a grandmother of x."

$$G(Helen) =$$

$$G(Jane) =$$

$$G(lan) =$$

$$G(Helen) =$$

$$G(Jane) =$$

$$G(lan) =$$

$$G(Guy) =$$

$$G(Guy) =$$

Relations in families pair people with other people. Relations in algebra pair numbers with other numbers. Here are some relations which involve numbers:

The "greater than" relation: We will use G(x) to mean "a number greater than x."

$$G(5) = 7$$
 means "A number greater than 5 is 7." or $7 > 5$

$$G(5) = 10.4$$
 means "A number greater than 5 is 10.4." or 10.4 > 5

Name some other numbers which can be paired up with 5 in this relation:

$$G(5) =$$

$$G(5) =$$

$$G(5) =$$

$$G(5) =$$

Find a number to make each of these true:

$$G(2) =$$

$$G(^{-}6) =$$

$$G(100) =$$

$$G(4.8) =$$

The "less than" relation: We will use L(x) to mean "a number less than x."

$$L(3) = 2$$
 means "A number less than 3 is 2." or 2 < 3

$$L(3) = -\frac{1}{3}$$
 means "A number less than 3 is $-\frac{1}{3}$." or $-\frac{1}{3} < 3$

$$L(^-1) = means$$

$$L(-2.5) = means$$

The "equality" relation: We will use E(x) to mean "a number equal to x."

$$E(2) = 2$$
 means "A number equal to 2 is 2." or $2 = 2$

$$E(^-6) = means$$

$$E(11.2) = means$$

The "less than or equal to" relation: We will use T(x) to mean "a number less than or equal to x."

$$T(4) = O$$
 means "A number less than or equal to 4 is 0." or $O \le 4$

$$T\left(\frac{3}{5}\right) = \frac{3}{5}$$
 means

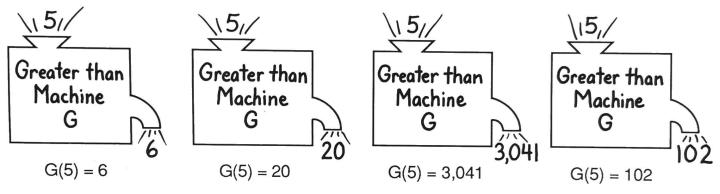
$$T(-6.3) = means$$

Functions

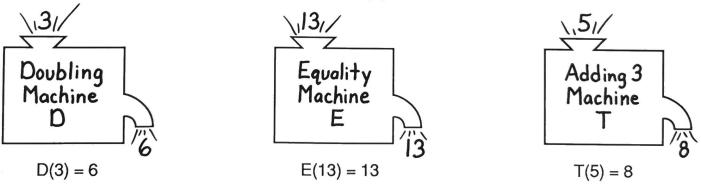
Some people like to think of relations as "input-output" machines.



As you can see, the output of a "greater than" machine can't be predicted. All we know is that it will put out a number greater than the number which is put in. Here are some possibilities:



Some kinds of relation machines are completely predictable.

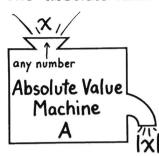


These completely predictable relations are called **functions**. A function has only one

possible output for each input.

Here are some more examples of functions.

The "absolute value" function: A(x) will mean "the absolute value of x." A(x) = |x|



$$A(7) = |7| = 7$$

$$A(-5.3) = |-5.3| = 5.3$$

$$A(-7) = |-7| = 7$$

$$A(4.71) = |4.71| = 4.71$$

$$A(3) =$$

$$A\left(\frac{1}{2}\right) =$$

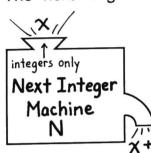
$$A(^{-}3) =$$

$$A\left(\frac{-3}{4}\right) =$$

$$A(^{-}12) =$$

$$A(^{-}6.6) =$$

The "next integer" function: N(x) will mean "the integer after x." N(x) = x + 1



$$N(10) = 11$$

$$N(-16) = -15$$

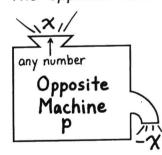
$$N(9) =$$

$$N(-101) =$$

$$N(0) =$$

$$N(25) =$$

The "opposite" function: P(x) will mean "the opposite of x." P(x) = -x



$$P(2) = -2 P\left(\frac{3}{5}\right) =$$

$$P\left(\frac{3}{5}\right) =$$

$$P(3.5) =$$

$$P(\bar{}1) = P\left(\frac{1}{2}\right) =$$

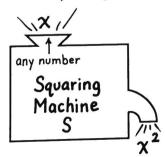
$$P(0.6) =$$

$$P(0) =$$

$$P\left(\frac{-10}{7}\right) =$$

$$P(-0.003) =$$

The "squaring" function: S(x) will mean "the square of x." $S(x) = x^2$



$$S(3) = 9$$

$$S(^{-}3) =$$

$$S(12) =$$

$$S(4) =$$

$$S(^{-}4) =$$

$$S(100) =$$

$$S(5) =$$

$$S(-5) =$$

$$S(-100) =$$

$$S(6) =$$

$$S(^{-}6) =$$

$$S(0.5) =$$

Here are some functions we use in everyday life:

The "first class postage" function: P(x) is the first class postage on a letter weighing x ounces. It costs 25¢ to mail a letter weighing one ounce or less. For each additional ounce or part of an ounce you pay 20¢ more.

$$P(1) =$$

$$P\left(\frac{1}{4}\right) =$$

$$P(0.7) =$$

$$P(2) =$$

$$P\left(\frac{3}{4}\right) =$$

$$P(7) =$$

$$P(3) =$$

$$P\left(1\frac{1}{4}\right) =$$

$$P(7.3) =$$

$$P(4) =$$

$$P\left(2\frac{1}{4}\right) =$$

$$P(3.7) =$$

$$P(5) =$$

$$P\left(3\frac{1}{2}\right) =$$

$$P(4.9) =$$

The "feet-to-inches" function: I(x) is the number of inches in x feet. There are 12 inches in one foot, so I(1) = 12.

$$I(2) =$$

$$I(5) =$$

$$I\left(\frac{1}{2}\right) =$$

$$I(3) =$$

$$I(9) =$$

$$I\left(1\frac{1}{2}\right) =$$

$$I(4) =$$

$$I(15) =$$

$$I\left(\frac{1}{3}\right) =$$

The "sales tax" function: S(x) is the sales tax on a taxable purchase of x dollars. To figure these out, you need to know the sales tax percentage for your state. Try to get a copy of the sales tax table which many stores use.

$$S(.10) =$$

$$S(.50) =$$

$$S(.87) =$$

$$S(.20) =$$

$$S(1.00) =$$

$$S(1.10) =$$

$$S(.30) =$$

$$S(2.00) =$$

$$S(7.75) =$$

The "days-in-a-year" function: D(x) is the number of days in the year x. Leap years have 366 days. Others have 365 days.

$$D(1980) =$$

$$D(1971) =$$

$$D(1776) =$$

$$D(1950) =$$

$$D(1900) =$$

$$D(2000) =$$

Instead of describing a function in words, we often define it by giving an algebraic expression for the number which is paired up with x. You finish this example:

The "f" function: f(x) is one more than 3 times x. f(x) = 3x + 1

$$f(5) = 3(5) + 1 = 15 + 1 = 16$$
 $f(8) =$

$$f(8) =$$

$$f(-2) = 3(-2) + 1 = -6 + 1 = -5$$
 $f(-8) =$

$$f(-8) =$$

Each function below is defined by an algebraic expression. Find the numbers asked for by substituting in the expression.

$$g(x) = 3x - 1$$

$$q^{(-4)} = 3(-4) - 1 = -12 - 1 = -13$$

$$q(7) = 3(7) - 1 =$$

$$g(5) =$$

$$h(x) = 3(x-1)$$

$$h(-4) =$$

$$h(7) =$$

$$h(5) =$$

$$p(x) = x^2 + 4$$

$$p(3) =$$

$$p(1) =$$

$$p(-10) =$$

$$q(x) = (x + 4)^2$$

$$q(3) =$$

$$q(1) =$$

$$r(x) = |x+6|$$

$$r(3) =$$

$$r(-10) =$$

$$r(^{-}6) =$$

$$s(x) = |x| + 6$$

$$s(3) =$$

$$s(-10) =$$

$$s(^{-}6) =$$

$$m(x) = x - 1$$

$$m(8) =$$

$$m(-8) =$$

$$m(O) =$$

$$n(x) = 1 - x$$

$$n(8) =$$

$$n(-8) =$$

$$n(0) =$$

A table is an easy way to list the pairs of numbers that belong to a function. The expression for the function is written above the table. The first column of the table lists the numbers to be substituted for the variable in the function. The second column lists the values obtained by substituting the numbers in the first column.

f(x) = x - 10		
*	f(x)	
25	25 - 10 = 15	
17	17-10=7	
12		
10		
8		
2		
0		
-3		

g(x) = 2x + 6		
χ.	g(x)	
-6		
-2		
-1		
0		
1	-	
2		
6		
100		

g(x)	= 2x + 6		h(x)	= O·x
χ	g(x)		X	h(x)
-6		,	-90	
-2			-10	
-1		,	-2	
0			-1	
			0	
2			1	
6			15	
100			40	
		•		

$k(x) = \frac{x}{6}$		
×	k(x)	
0		
18		
-18		
60		
-60		
ı		
-1		
5		

m(x) =	2
χ	m(x)
-3	
-2	
-1	
0	
I	
2	
3	
4	

$n(x) = \frac{x-5}{4}$		
χ	n(x)	
-3		
-2		
-1		
0		
1		
2		
3		
4		

Many functions important in science and other fields have been discovered by people who collected data, organized their information in a table, and figured out a **formula** for it.

See if you can find a formula to fit the data in each table.

f(x) =		
χ	$f(\chi)$	
5	15	
2	6	
1	3	
0	0	
-(-3	
-2	-6	
-5	⁻ 15	

$$g(x) = x$$
 $g(x)$
 10
 12
 5
 7
 3
 5
 0
 2
 -3
 -1
 -5
 -3
 -15
 -13

p(x) =	:
×	p(x)
-20	20
-12	12
-6	6
0	0
6	-6
12	-12
20	-20

q(x) =	
χ	q(x)
4	9
2	5
1	3
0	1
-1	-1
-2	-3
-4	-7

$$r(x) = \frac{r(x)}{3} = \frac{r(x)}{3} = \frac{1}{9} = \frac{1}{1} = \frac$$

$s(\chi) =$	
x	s(x)
5	-10
3	-6
- 1	-2
-1	2
3	6
5	10
-7	14

12	-12	
20	-20	
V(X) =		
٧	v(x)	
7	7	
4	4	
0	0	
-2	2	
-5	5	
-8	8	
-60	60	
01990 by Key Curriculum Project, Inc.		

Written Work

Do these problems on some clean paper. Label each page of your work with your name, your class, the date, and the book number. Also number each problem. Keep this written work inside your book, and turn it in with your book when you are finished.

Write each rational number as a fraction.

-7

- $2\frac{1}{2}$
- 0
- 0.6
- $^{-}4.3$
- 1.04
- Write the answer to the division problem $^{-}15 \div 4$ in three different ways. 2.
- Solve each equation. Write your answer as an integer or a mixed number. 3.

-9x = 95

$$\frac{5x}{6} = 14$$

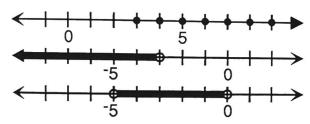
$$\frac{6x-2}{3} = 7$$

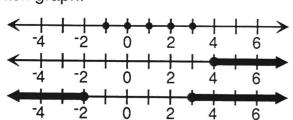
$$x(x + 3) = x^2 - 16$$

Name each point which is marked.

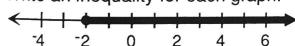


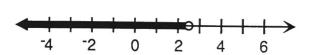
- In words, describe the set of numbers shown by each graph.





Write an inequality for each graph.





Solve each equation or inequality.

$$|x+1|=4$$

$$3x - 2 \le 19$$

$$\frac{-4x}{3}$$
 < 20

$$|x-5| = 0$$

$$6 - 5x > 2$$

$$6 - 5x > 2$$
 $7x - 1 > 4 - 3x$

$$|x+4| < 12$$

$$3|x| - 7 = 11$$

$$3(x-2) \le 4x + 16$$

Which of these relations are functions?

The "brother" relation: B(x) is a brother of x.

The "mother" relation: M(x) is a mother of x.

The "less than" relation: L(x) is a number less than x.

The "2 less than" relation: W(x) = x - 2.

The "tripling" relation: T(x) = 3x.

Make a table for each function below. Choose five numbers to substitute for x in each table. Show the values you get when you substitute these numbers.

$$f(x) = 2x - 1$$

$$g(x) = |x-2|$$

$$h(x) = x^2$$

Practice Test

Write each rational number as a fraction.

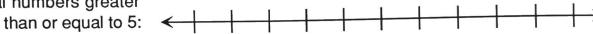
$$3\frac{4}{5} = -2\frac{1}{9} = 0.7 = 4.01 =$$

Draw a number line showing numbers from $^{-3}$ to 3. Graph the numbers $^{-2.5}$, 0.3 and $2\frac{1}{3}$.

Label each number line and graph the set of numbers.



Rational numbers greater



Rational numbers not



Find each absolute value.

Solve.

$$|x| = 7$$

$$|x-5|=2$$

$$|x|+4=1$$

$$2x + 3 > 11$$

$$x - 5 < -9$$

$$-6x \ge 48$$

Solve.

$$4x - 7 < x + 8 \qquad 9 - x \ge -3 \qquad \frac{x}{2} < -9$$

$$20 < 5x - 2 \qquad 7x - 3x + 2 > x + 5x - 6 \qquad \frac{2x}{5} + 6 \le 4$$

G(x) means "a number greater than x." Find a number to make each of these true.

$$G(4) =$$

$$G_{1}(-2.3) =$$

$$G(0) =$$

Fill in the missing numbers in the table for each function.

$$g(x) = x^{2} + 1$$
 x
 $g(x)$

20

4

1

0

-1

-3

-10