Key to

Algebra

6
Student Workbook

Multiplying and Dividing Rational Expressions

By Julie King and Peter Rasmussen

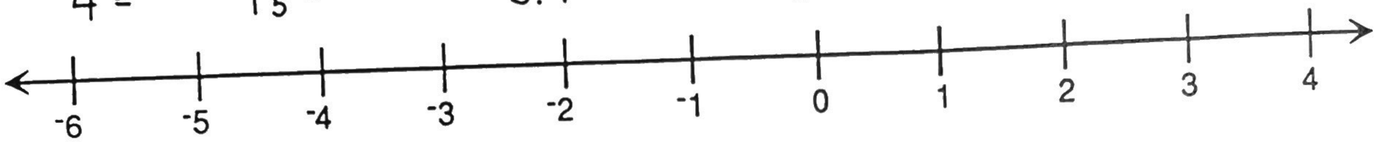
Name Class

Review

Remember that a rational number can always be written as a fraction with integers as the numerator and denominator. In this book we will study algebraic expressions that can stand for rational numbers. You will have to use what you know about polynomials as well as what you know about rational numbers.

Write each rational number as a fraction. Then graph it on the number line.

$-4 =$ $\quad 1\frac{2}{5} =$ $\quad 3.4 =$ $\quad -2\frac{1}{2} =$ $\quad -4.9 =$ $\quad 0 =$



Factor 36 five different ways.

36 $\quad$ 36 $\quad$ 36 $\quad$ 36 $\quad$ 36

Write an equivalent expression using exponents.

$5 \cdot xxxx =$

$7y \cdot 7y \cdot 7y =$

$rr \cdot 2t \cdot 2t \cdot 2t =$

$3n \cdot 5n =$

$p \cdot 4p \cdot 2p =$

$(xx)(yyy) =$

Multiply.

$c^8 \cdot c =$

$a^2 \cdot a^5 =$

$2y^4 \cdot 6y^3 =$

$6(p-5) =$

$x(x+4) =$

$5h(h+3) =$

$(x+2)(x+6) =$

$(2x+3)(3x-1) =$

Factor.

$3x - 12 =$

$8y - 12 =$

$a^2 - 81 =$

$x^2 + 6x + 9 =$

$t^2 - 11t + 10 =$

$3n^2 - n - 4 =$

Solve each equation.

$\frac{x}{5} = 30$ $\qquad \frac{5x}{12} = 15$ $\qquad \frac{x-4}{3} = 12$

Rational Expressions

Remember that a term is an algebraic expression in which multiplication is the only operation. A polynomial can be a single term or can be made by adding and subtracting terms.

Polynomials: $x^2 + 1$ $\quad 19 \quad a^2 - 4a + 5 \quad 3x^3 \quad {}^-5x + 2y$

A rational number is a fraction with *integers* for the numerator and denominator.
A **rational expression** is a fraction with *polynomials* for the numerator and denominator.

Rational Number: $\dfrac{\text{Integer}}{\text{Integer}}$ Rational Expression: $\dfrac{\text{Polynomial}}{\text{Polynomial}}$

Of course, the integer or polynomial on the bottom cannot be 0 (since we cannot divide by 0). Here are some examples of rational expressions:

$$\frac{7}{x} \qquad \frac{x-5}{x+5} \qquad \frac{y+3}{y^2-9} \qquad \frac{0}{n+8} \qquad \frac{x^2-4x+4}{x+6}$$

Make as many rational expressions as you can by using one of these polynomials for the numerator and one for the denominator.

$x^2 \qquad 10 \qquad x - 5 \qquad x^2 + 3x + 2$

Find the value of each rational expression when $x = 3$. If the answer is not an integer, leave it as a fraction.

$\frac{x+4}{x+7} = \frac{3+4}{3+7} = \frac{7}{10}$	$\frac{x}{x+2} =$	$\frac{6}{-x} =$
$\frac{x-5}{x+3} =$	$\frac{x-1}{x+1} =$	$\frac{x^2+1}{x+1} =$
$\frac{2x-6}{x-6} =$	$\frac{x^2-4}{x+2} =$	$\frac{10+x}{10-x} =$

$\frac{x-8}{x^2+2x+1} =$	$\frac{x^2-5x+10}{x-5} =$
$\frac{x^2+4x+4}{x^2-4x+4} =$	$\frac{^-2x^2}{4x-1} =$

Find the value of each rational expression when $x = {}^-2$ and $y = 5$.

$\frac{x}{y} =$	$\frac{3x}{3y} =$	$\frac{y}{x} =$
$\frac{x+2}{y+2} =$	$\frac{x-2}{y-2} =$	$\frac{x+5}{y+5} =$
$\frac{1}{xy} =$	$\frac{xy}{1} =$	$\frac{x^2y^2}{xy} =$

$\frac{x}{2} + \frac{y}{5} =$	$x\left(\frac{y}{y-7}\right) =$

$\frac{x^2+2xy+y^2}{x+y} =$

You might have noticed that every answer on page 3 was a rational number. Once in a while we do *not* get a rational number for an answer when we replace variables with numbers in a rational expression. Look at the substitution table below.

a	$\frac{6}{a+5}$
4	$\frac{6}{4+5} = \frac{6}{9}$
1	$\frac{6}{1+5} = \frac{6}{6} = 1$
0	$\frac{6}{0+5} = \frac{6}{5}$
$^-5$	$\frac{6}{^-5+5} = \cancel{\frac{6}{0}}$

Do you see what happened when $a = {}^-5$? When $a = {}^-5$, $\frac{6}{a+5}$ has no value because it equals $6 \div 0$, which has no answer. We say that $\frac{6}{a+5}$ is **undefined** when $a = {}^-5$.

You finish these substitution tables. Be on the lookout for substitutions which give no answer. Write "undefined" whenever you find one.

x	$\frac{x+3}{2}$
3	
0	
$^-3$	
$^-6$	

n	$\frac{n+1}{n+4}$
4	
2	
0	
$^-4$	

r	$\frac{r}{r^2-9}$
0	
1	
3	
$^-3$	

b	c	$\frac{b}{c}$
5	$^-5$	
2	0	
0	2	
$^-2$	$^-1$	

To be sure that a rational expression always stands for a number, we have to set the **condition** that the denominator cannot equal zero.

$\frac{x}{y}$, $y \neq 0$ I can choose any number except 0 for y.

$\frac{a}{a+2}$, $a + 2 \neq 0$

$\frac{x^2+5x+6}{x^2-9}$, $x^2 - 9 \neq 0$

You write the condition for each fraction.

$\frac{a}{b}$, ________________ $\frac{5}{y}$, ________________

$\frac{x+y}{x-y}$, ________________ $\frac{2x}{3x-5}$, ________________

From now on we shall *assume* that the denominator of each fraction is not zero. That's so we won't have to write down the condition each time.

You already know that every integer can be written as a fraction with a denominator of 1. Every polynomial can also be written as a fraction with a denominator of 1. This means that every polynomial is a rational expression.

$x + 3 = \frac{x+3}{1}$ $a^2 - 6 = \frac{a^2-6}{1}$ $y^2 + 3y - 5 = \frac{y^2+3y-5}{1}$

Write each polynomial as a fraction.

$5 - x =$

$14 - y^2 =$

$x^2 + 4x =$

$3a^4 =$

$a^2 - 2 =$

$x^3 + x^2 =$

$n^2 + 7n + 12 =$

$5xy =$

$5c^2 + 10 =$

$x^2 + 7x + 10 =$

Multiplying Fractions

Multiplying fractions is easy, whether they are numbers or expressions. You just multiply the numerators and multiply the denominators.

Multiply.

$\frac{5}{3} \cdot \frac{7}{2} = \frac{35}{6}$	$\frac{4}{11} \cdot \frac{5}{3} =$	$\frac{-6}{7} \cdot \frac{1}{8} =$	$\frac{9}{10} \cdot \frac{3}{2} =$
$\frac{5}{6} \cdot \frac{-2}{7} =$	$\frac{-3}{5} \cdot \frac{-3}{5} =$	$\frac{2}{3} \cdot \frac{4}{3} \cdot \frac{2}{5} =$	$\frac{-1}{5} \cdot \frac{3}{2} \cdot \frac{3}{4} =$
$\frac{x}{3y} \cdot \frac{4}{3y} = \frac{4x}{9y^2}$	$\frac{a}{b} \cdot \frac{a}{b} =$	$\frac{3}{7} \cdot \frac{x}{y} =$	$\frac{4}{a} \cdot \frac{b}{9} =$
$\frac{10x}{7} \cdot \frac{2x}{3} =$	$\frac{3x^2}{5y} \cdot \frac{x^3}{2y^2} =$	$\frac{5x^3}{3y^5} \cdot \frac{7x^2}{8y} =$	$\frac{2x}{3y} \cdot \frac{2x}{3y} \cdot \frac{2x}{3y} =$

Multiplying a fraction by an integer or a polynomial is easy, too. Write the integer or the polynomial as a fraction by putting a 1 on the bottom. Then go ahead and multiply numerators and denominators.

$\frac{3}{1} \cdot \frac{2}{5} = \frac{6}{5}$	$-5 \cdot \frac{3}{4} =$	$6 \cdot \frac{-7}{5} =$	$\frac{1}{5} \cdot 6 =$
$x \cdot \frac{x}{y} =$	$5 \cdot \frac{x}{2} =$	$x \cdot \frac{y}{4} =$	$\frac{3}{5} \cdot x =$
$3x^2 \cdot \frac{x^2}{y^2} =$	$\frac{5a}{3b} \cdot 5a^3 =$	$x^3 \cdot \frac{x^2}{y} =$	$\frac{2a}{7b} \cdot 3c =$

Some multiplication problems look hard but are easy if you change each number to a fraction.

$2\frac{5}{8} \cdot 5\frac{1}{3} = \frac{\cancel{21}^{7}}{\cancel{8}_{1}} \cdot \frac{\cancel{16}^{2}}{\cancel{3}_{1}} = \frac{14}{1} = 14$	$3\frac{4}{5} \cdot 2\frac{1}{4} =$
$5\frac{4}{7} \cdot 1\frac{1}{13} =$	$-1\frac{5}{7} \cdot 1\frac{3}{4} =$
$(3.5a)(\frac{2}{7}b) =$	$(2\frac{1}{3})(.09x^2) =$

Multiply.

$\frac{3}{x+5} \cdot \frac{x-2}{x} = \frac{3(x-2)}{x(x+5)} = \frac{3x-6}{x^2+5x}$

$\frac{x}{x-8} \cdot \frac{x+5}{6} =$

$\frac{7}{x+4} \cdot \frac{x-4}{2x} =$

$\frac{3}{4} \cdot \frac{x+5}{x+7} =$

$\frac{x}{y} \cdot \frac{x+y}{x-y} =$

$\frac{2a+3}{3a+4} \cdot \frac{2}{3} =$

$\frac{5}{x+3} \cdot (x-4) =$

$(a+5) \cdot \frac{a}{a-2} =$

$\frac{x-3}{2x} \cdot \frac{x-6}{3x} = \frac{(x-3)(x-6)}{2x \cdot 3x} = \frac{x^2-9x+18}{6x^2}$

$\frac{x+3}{x} \cdot \frac{x+4}{x} =$

$\frac{x+5}{x-6} \cdot \frac{x-6}{x+5} =$

$\frac{a-3}{a-7} \cdot \frac{a+3}{a+7} =$

$(x+6) \cdot \frac{x-2}{x-3} =$

$\frac{y-5}{y+5} \cdot (y-5) =$

Here are some rational expressions which have exponents. Write each one out the long way. Then multiply.

$\left(\frac{2}{5}\right)^2 = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$

$\left(\frac{3}{4}\right)^2 =$

$\left(\frac{-1}{6}\right)^2 =$

$\left(\frac{x}{y}\right)^2 =$

$\left(\frac{2x}{9}\right)^2 =$

$\left(\frac{3a}{10}\right)^2 =$

$\left(\frac{-2}{3}\right)^3 =$

$\left(\frac{-2}{3}\right)^4 =$

$\left(\frac{x}{y}\right)^4 =$

$\left(\frac{3a}{10}\right)^3 =$

$\left(\frac{x-5}{x+6}\right)^2 = \frac{x-5}{x+6} \cdot \frac{x-5}{x+6} = \frac{x^2-10x+25}{x^2+12x+36}$

$\left(\frac{x+2}{x+4}\right)^2 =$

$\left(\frac{x-3}{x-7}\right)^2 =$

$\left(\frac{5}{x+5}\right)^2 =$

$\left(\frac{2x+1}{x-9}\right)^2 =$

Equivalent Fractions

Every fraction that has the same numerator and denominator (except $\frac{0}{0}$) is equal to 1. That's because the denominator goes into the numerator one time. Each fraction below is equal to 1.

$$\frac{2}{2} \qquad \frac{3}{3} \qquad \frac{4}{4} \qquad \frac{8}{8} \qquad \frac{100}{100} \qquad \frac{^-1}{^-1} \qquad \frac{^-2}{^-2} \qquad \frac{^-5}{^-5} \qquad \frac{^-50}{^-50}$$

When we multiply a number by 1, we always end up with a number equal to the number we started with. For example:

$$5 \cdot 1 = 5 \qquad\qquad {}^-6 \cdot 1 = {}^-6 \qquad\qquad 342 \;\; 1 = 342$$

The same thing happens when we multiply a fraction by another fraction that's equal to 1. We end up with a fraction that's **equivalent** to the fraction we started with. Here are some examples:

$$\frac{1}{2} \cdot \boxed{\frac{2}{2}} = \frac{2}{4} \qquad\qquad \frac{1}{2} \cdot \boxed{\frac{3}{3}} = \frac{3}{6} \qquad\qquad \frac{1}{2} \cdot \boxed{\frac{4}{4}} = \frac{4}{8}$$

equivalent fractions   equivalent fractions   equivalent fractions

$\frac{2}{4}$, $\frac{3}{6}$ and $\frac{4}{8}$ are all equivalent to $\frac{1}{2}$. Find some more fractions equivalent to $\frac{1}{2}$.

$$\frac{1}{2} \cdot \frac{5}{5} = \qquad \frac{1}{2} \cdot \frac{6}{6} = \qquad \frac{1}{2} \cdot \frac{7}{7} = \qquad \frac{1}{2} \cdot \frac{8}{8} =$$

$$\frac{1}{2} \cdot \frac{9}{9} = \qquad \frac{1}{2} \cdot \frac{^-9}{^-9} = \qquad \frac{1}{2} \cdot \frac{50}{50} = \qquad \frac{1}{2} \cdot \text{—} =$$

(You pick the number.)

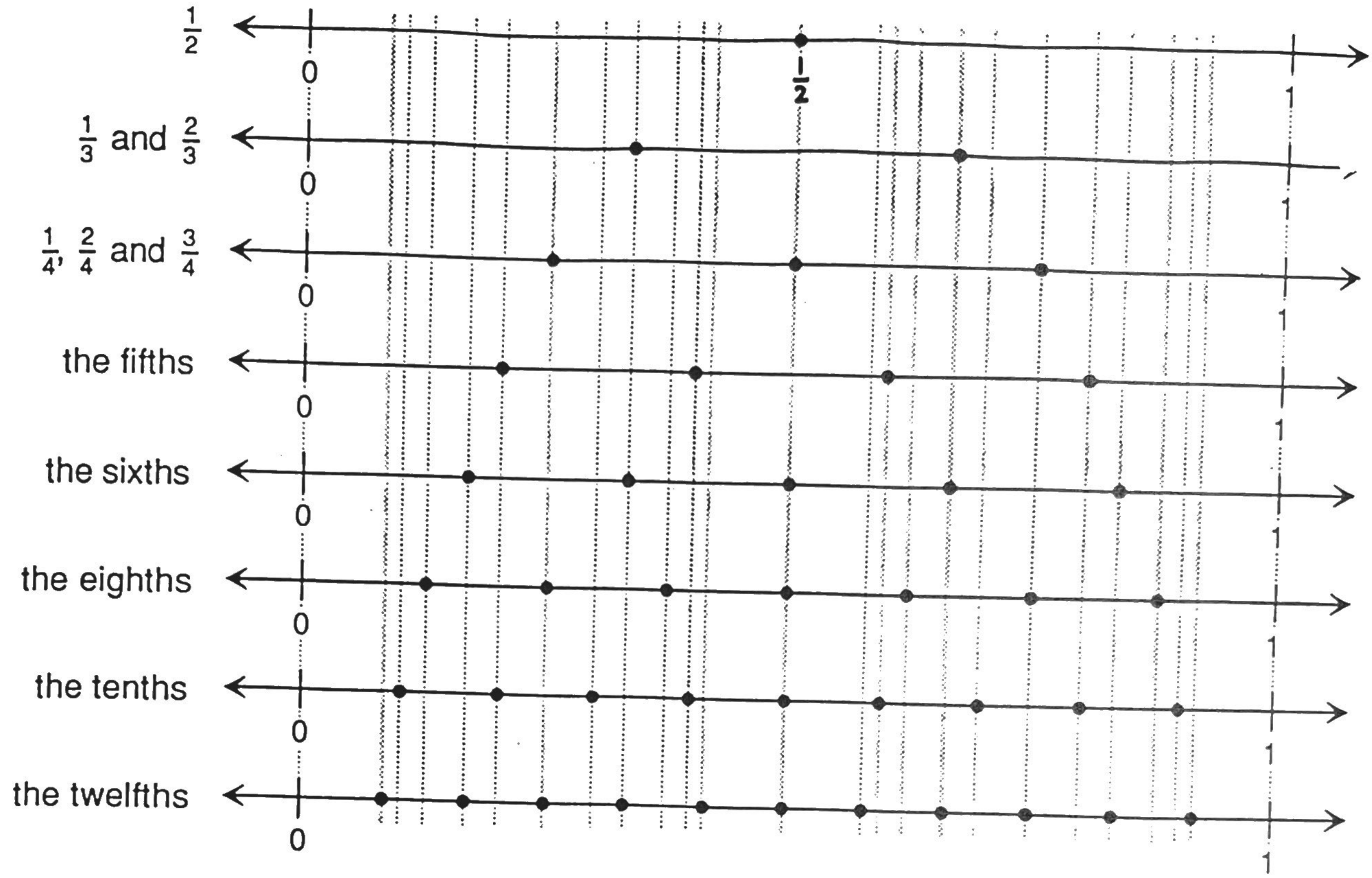
Find some fractions equivalent to $\frac{3}{4}$.

$$\frac{3}{4} \cdot \frac{2}{2} = \qquad \frac{3}{4} \cdot \frac{3}{3} = \qquad \frac{3}{4} \cdot \frac{4}{4} = \qquad \frac{3}{4} \cdot \frac{^-5}{^-5} =$$

$$\frac{3}{4} \cdot \text{—} = \qquad \frac{3}{4} \cdot \text{—} = \qquad \frac{3}{4} \cdot \text{—} = \qquad \frac{3}{4} \cdot \text{—} =$$

(You pick numbers for these.)

Label the points shown on each number line below.



List all the fractions whose graphs are directly below the graph of $\frac{1}{2}$:

What is true about all of these fractions?

List all the fractions shown above that are equivalent to each fraction below.

$\frac{1}{3}$:	$\frac{3}{4}$:	$\frac{4}{5}$:
$\frac{2}{3}$:	$\frac{1}{5}$:	$\frac{1}{6}$:
$\frac{1}{4}$:	$\frac{2}{5}$:	$\frac{2}{6}$:
$\frac{2}{4}$:	$\frac{3}{5}$:	$\frac{5}{6}$:

Rewriting Fractions in Higher Terms

As you can see, there is an easy way to rewrite a fraction in higher terms. We just pick a number or expression which is not 0 and multiply the numerator and denominator of the fraction by the number or expression we picked. That amounts to multiplying the fraction by 1, so the answer comes out equivalent to the fraction we started with.

Pick 3: $\frac{7}{8} = \frac{7 \cdot 3}{8 \cdot 3} = \frac{21}{24}$

We ended up with $\frac{21}{24}$, which is equivalent to $\frac{7}{8}$ and is in **higher terms.**

For each fraction, find an equivalent fraction in higher terms.

Multiply numerator and denominator

by 5:	$\frac{5 \cdot 5}{6 \cdot 5} = \frac{25}{30}$	$\frac{-2 \cdot 5}{7 \cdot 5} =$	$\frac{x \cdot 5}{3 \cdot 5} =$
by 2:	$\frac{9}{10} =$	$\frac{-1}{3} =$	$\frac{4}{y} =$
by -3:	$\frac{7}{2} =$	$\frac{-4}{5} =$	$\frac{x-1}{6} =$
by x:	$\frac{3}{8} =$	$\frac{1}{6} =$	$\frac{2x}{3y} =$
by a:	$\frac{9}{5} =$	$\frac{-1}{a} =$	$\frac{a^2}{a+4} =$
by $2n$:	$\frac{1}{8} =$	$\frac{3n^2}{m} =$	$\frac{n+1}{n^2} =$
by $-4b$:	$\frac{2}{3} =$	$\frac{5}{b} =$	$\frac{1-b}{b} =$

To rewrite a fraction in higher terms, multiply the numerator and denominator by the same number or polynomial.

Find five fractions equivalent to $\frac{2}{5}$.

$\frac{2 \cdot 2}{5 \cdot 2} = \frac{4}{10}$

$\frac{2 \cdot 3}{5 \cdot 3} =$

$\frac{2 \cdot 5}{5 \cdot 5} =$

$\frac{2 \cdot 10}{5 \cdot 10} =$

Find five fractions equivalent to $\frac{1}{4}$.

$\frac{1 \cdot 2}{4 \cdot 2} =$

$\frac{1 \cdot}{4 \cdot} =$

$\frac{1 \cdot}{4 \cdot} =$

$\frac{1 \cdot}{4 \cdot} =$

Find five fractions equivalent to $\frac{3}{8}$.

$\frac{3 \cdot}{8 \cdot} =$

$\frac{3 \cdot}{8 \cdot} =$

$\frac{3 \cdot}{8 \cdot} =$

$\frac{3 \cdot}{8 \cdot} =$

Find five fractions equivalent to $\frac{2x}{3}$.

$\frac{2x \cdot 2}{3 \cdot 2} = \frac{4x}{6}$

$\frac{2x \cdot 5}{3 \cdot 5} =$

$\frac{2x \cdot x}{3 \cdot x} =$

$\frac{2x \cdot 3x}{3 \cdot 3x} =$

$\frac{2x \cdot y}{3 \cdot y} =$

Find five fractions equivalent to $\frac{-5}{x}$.

$\frac{-5 \cdot 2}{x \cdot 2} =$

$\frac{-5 \cdot 10}{x \cdot 10} =$

$\frac{-5 \cdot x}{x \cdot x} =$

$\frac{-5 \cdot (x+4)}{x \cdot (x+4)} =$

$\frac{-5 \cdot (x-1)}{x \cdot (x-1)} =$

Find five fractions equivalent to $\frac{x}{y}$.

$\frac{x \cdot 2}{y \cdot 2} =$

$\frac{x \cdot -6}{y \cdot -6} =$

$\frac{x \cdot x}{y \cdot x} =$

$\frac{x \cdot (x+1)}{y \cdot (x+1)} =$

$\frac{x \cdot (x+2)}{y \cdot (x+2)} =$

Find 5 fractions equivalent to $\frac{x+3}{x-2}$.

$\frac{(x+3) \cdot 2}{(x-2) \cdot 2} = \frac{2x+6}{2x-4}$

$\frac{(x+3) \cdot 5}{(x-2) \cdot 5} =$

$\frac{(x+3) \cdot x}{(x-2) \cdot x} =$

$\frac{(x+3) \cdot 3x}{(x-2) \cdot 3x} =$

$\frac{(x+3)\,(x+4)}{(x-2)\,(x+4)} =$

Find 5 fractions equivalent to $\frac{x-6}{x+4}$.

$\frac{(x-6) \cdot 2}{(x+4) \cdot 2} =$

$\frac{(x-6) \cdot x}{(x+4) \cdot x} =$

$\frac{(x-6)\,(x-1)}{(x+4)\,(x-1)} =$

$\frac{(x-6)\,(x+3)}{(x+4)\,(x+3)} =$

$\frac{(x-6)\,(x+6)}{(x+4)\,(x+6)} =$

If we want an equivalent fraction with a certain denominator, we must figure out what to multiply the top and bottom by to get that denominator Here are two examples:

$\frac{2}{3} = \frac{\quad}{24}$ 24 is 3 times 8, so I should also multiply 2 by 8.

$\frac{6}{x} = \frac{\quad}{3x^2}$ $3x^2$ is $x \cdot 3x$, so I should also multiply 6 by $3x$.

$\frac{2\cdot 8}{3\cdot 8} = \frac{16}{24}$

$\frac{6\cdot 3x}{x\cdot 3x} = \frac{18x}{3x^2}$

You try these. First figure out what the denominator has been multiplied by. Then multiply the numerator by the same number or expression.

$\frac{4\cdot 10}{5\cdot 10} = \frac{40}{50}$	$\frac{-3\,\cdot}{5\,\cdot} = \frac{\quad}{35}$	$\frac{7\,\cdot}{2\,\cdot} = \frac{\quad}{50}$
$\frac{-1\,\cdot}{9\,\cdot} = \frac{\quad}{27}$	$\frac{5\,\cdot}{-8\,\cdot} = \frac{\quad}{32}$	$\frac{-4\,\cdot}{3\,\cdot} = \frac{\quad}{63}$
$\frac{x\cdot 6}{y\cdot 6} = \frac{6x}{6y}$	$\frac{2t\,\cdot}{3\,\cdot} = \frac{\quad}{12}$	$\frac{n\,\cdot}{2m\,\cdot} = \frac{\quad}{10m}$
$\frac{2a\,\cdot}{b\,\cdot} = \frac{\quad}{b^2}$	$\frac{5p\,\cdot}{4\,\cdot} = \frac{\quad}{4p^2}$	$\frac{r\,\cdot}{-2r\,\cdot} = \frac{\quad}{8r^2}$
$\frac{(x+5)\,\cdot}{4\,\cdot} = \frac{\quad}{28}$	$\frac{3\,\cdot}{(x-1)\,\cdot} = \frac{\quad}{5(x-1)}$	$\frac{(x+1)\,\cdot}{(x+2)\,\cdot} = \frac{\quad}{6(x+2)}$

In these problems you will have to factor the new denominator to find out what to multiply by.

3x−6 equals 3(x−2), so I should multiply the top and bottom by 3.

$\frac{7\cdot 3}{(x-2)\cdot 3} = \frac{21}{\cancel{3x-6}\ \ 3(x-2)}$	$\frac{6\,\cdot}{(x-9)\,\cdot} = \frac{\quad}{2x-18}$
$\frac{5\,\cdot}{(x+4)\,\cdot} = \frac{\quad}{10x+40}$	$\frac{1\,\cdot}{(3x+5)\,\cdot} = \frac{\quad}{12x+20}$
$\frac{x\,\cdot}{(x-3)\,\cdot} = \frac{\quad}{x^2-3x}$	$\frac{2n\,\cdot}{(n+1)\,\cdot} = \frac{\quad}{n^2+n}$
$\frac{2\,\cdot}{(x+5)\,\cdot} = \frac{\quad}{x^2+7x+10}$	$\frac{3\,\cdot}{(y-4)\,\cdot} = \frac{\quad}{y^2-16}$

Simplifying Fractions

We know how to rewrite a fraction in higher terms. We just multiply the numerator and denominator by the same number.

$$\frac{4^{\cdot 5}}{7_{\cdot 5}} = \frac{20}{35}$$

rewritten in higher terms

To **simplify** a fraction, or rewrite it in **lower terms**, we do the opposite. We factor the numerator and denominator of the fraction so that one of the factors on top is the same as one of the factors on the bottom. Then we cancel the same factor from the top and bottom. What is left is our simplified fraction, which will be equivalent to the fraction we started with. Here is an example:

5 goes into both 20 and 35.

$$\frac{20}{35} = \frac{5 \cdot 4}{5 \cdot 7} = \frac{\cancel{5}}{\cancel{5}} \cdot \frac{4}{7} = \frac{4}{7}$$

rewritten in lower terms

Simplify each fraction. Do each problem in two steps.

4 goes into 8 and 12. $\frac{8}{12} = \frac{\cancel{4} \cdot 2}{\cancel{4} \cdot 3} = \frac{2}{3}$	3 is a factor of $^-3$ and 12. $\frac{-3}{12} = \frac{\cancel{3} \cdot -1}{\cancel{3} \cdot 4} =$
$\frac{9}{12} =$	$\frac{-15}{25} =$
$\frac{6}{18} =$	$\frac{-4}{14} =$
$\frac{-12}{21} =$	$\frac{15}{6} =$
$\frac{20}{22} =$	$\frac{18}{24} =$
$\frac{-4}{16} =$	$\frac{-80}{30} =$

Simplify each rational expression by factoring the top and bottom and canceling the common factors.

2 is the only factor left on the top. On the bottom I have 3, y and y. $3\cdot y\cdot y = 3y^2$.	To keep track of the factors that are left, I circled them.
$\frac{6y^3}{9y^5} = \frac{2\cdot\cancel{3}\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}}{3\cdot\cancel{3}\cdot\cancel{y}\cdot\cancel{y}\cdot\cancel{y}\cdot y\cdot y} = \frac{2}{3y^2}$	$\frac{10x^3y^2}{35xy^3} = \frac{2\cdot\cancel{5}\cdot\cancel{x}\cdot x\cdot x\cdot\cancel{y}\cdot\cancel{y}}{7\cdot\cancel{5}\cdot\cancel{x}\cdot y\cdot\cancel{y}\cdot\cancel{y}} = \frac{2x^2}{7y}$
$\frac{20xy}{5x^2} =$	$\frac{12a^4}{10a^3} =$
$\frac{15x^5}{25x^2} =$	$\frac{-9x^2}{6x^2} =$
$\frac{3a^2}{a^5} =$	$\frac{x^2}{xy} =$
$\frac{10a^2}{40a} =$	$\frac{5ab}{a^3b} =$
$\frac{15x^5}{13x^2} =$	$\frac{x^2y^2}{x^2yz} =$
$\frac{9x^2y}{10y^2} =$	$\frac{x^4y}{xy^4} =$
$\frac{3x^2y^2}{21xy} =$	$\frac{20x^2y}{30xy^2} =$
$\frac{27x^3y}{36xy^3} =$	$\frac{-60a^2b^2}{40ab} =$

The more factoring and canceling you can do in your head, the less you will have to write out each time you simplify a fraction.

5 goes into 10 and 35. 5 into 10 is 2 and 5 into 35 is 7.

The x on the bottom cancels one of the x's on the top. So x^2 is left on the top.

The y^2 on top cancels two of the y's on the bottom. So y is left on the bottom.

$$\frac{\overset{2}{\cancel{10}}\,\overset{x^2}{\cancel{x^3}}\,\cancel{y^2}}{\underset{7}{\cancel{35}}\,\cancel{x}\,\underset{y}{\cancel{y^3}}} = \frac{2x^2}{7y}$$

Here are some more fractions for you to simplify. Try to do the canceling without writing out all the factors.

$\frac{6y^3}{9y^5} =$	$\frac{20xy}{5x^2} =$	$\frac{12a^4}{10a^3} =$
$\frac{15x^5}{25x^2} =$	$\frac{-9x^2}{6x^2} =$	$\frac{3a^2}{a^5} =$
$\frac{x^2}{xy} =$	$\frac{9x^2y}{10y^2} =$	$\frac{x^4y}{xy^4} =$
$\frac{3x^2y^2}{21xy} =$	$\frac{20x^2y}{30xy^2} =$	$\frac{27x^3y}{36xy^3} =$
$\frac{27x^3y^3}{36x^3y^3} =$	$\frac{-60a^2b^2}{40ab} =$	$\frac{10a^2}{40a} =$
$\frac{ab}{a^3b} =$	$\frac{15x^5}{3x^2} =$	$\frac{x^2y}{x^2y} =$

Did you get the last three problems? If you're not sure, look at the next page.

Sometimes you can cancel out all the factors on the TOP . . .

ab is really 1ab. $\frac{\overset{1}{\cancel{a}\cancel{b}}}{\underset{a^2}{\cancel{a^3}\cancel{b}}} = \frac{1}{a^2}$	$\frac{a}{a^2} =$	$\frac{x}{5x} =$
$\frac{3x}{9x} =$	$\frac{xy}{x^2y^2} =$	$\frac{2x^2}{10x^2y} =$

Sometimes you can cancel out all the factors on the BOTTOM . . .

$\frac{\overset{5\ \ x^3}{\cancel{15}\cancel{x^5}}}{\underset{1}{\cancel{3}\cancel{x^2}}} = \frac{5x^3}{1} = 5x^3$	$\frac{6x^3}{3x} =$	$\frac{5ab}{a} =$
$\frac{xy}{x} =$	$\frac{12x^3}{4x^2} =$	$\frac{3x^2y^2}{x^2y} =$

And sometimes you can cancel out all the factors on the TOP AND BOTTOM . . .

$\frac{\overset{1}{\cancel{x^2}\cancel{y}}}{\underset{1}{\cancel{x^2}\cancel{y}}} = \frac{1}{1} = 1$	$\frac{a}{a} =$	$\frac{-3x}{-3x} =$
$\frac{a^2b^2}{a^2b^2} =$	$\frac{4x^2}{4x^2} =$	$\frac{12xy}{12xy} =$

Simplify these rational expressions. Each one is already factored for you. You just have to cancel and write the answer.

$\frac{5x^2}{10x+15} = \frac{\cancel{5}\cdot x^2}{\cancel{5}(2x+3)} = \frac{x^2}{2x+3}$	$\frac{2a^2}{2a+10} = \frac{2\cdot a^2}{2(a+5)} =$
$\frac{4x+12}{8x-4y} = \frac{4(x+3)}{4(2x-y)} =$	$\frac{6x^2+9x}{3xy-12x^2} = \frac{3x(2x+3)}{3x(y-4x)} =$
$\frac{x^2+3x}{5x+15} = \frac{x\cancel{(x+3)}}{5\cancel{(x+3)}} =$	$\frac{x+4}{2x+8} = \frac{1(x+4)}{2(x+4)} =$
$\frac{3x-12}{x^2-4x} = \frac{3(x-4)}{x(x-4)} =$	$\frac{xy+2y}{7x+14} = \frac{y(x+2)}{7(x+2)} =$

You will have to factor the polynomials in each rational expression below. Then cancel factors and write your answer. Show each step.

$\frac{3y}{3y+6} = \frac{3\cdot}{3(\qquad)} =$	$\frac{x-6}{2x^2-12x} = \frac{1(\qquad)}{2x(\qquad)} =$
$\frac{x^2+2x}{5x+10} = \frac{x(\qquad)}{5(\qquad)} =$	$\frac{2x-8}{3x-12} = \frac{2(\qquad)}{3(\qquad)} =$
$\frac{x^2+5x}{6x+30} =$	$\frac{10x}{5x+40} =$
$\frac{x^2-9x}{x^2+xy} =$	$\frac{x^2+4x}{4x+16} =$

When we are simplifying rational expressions, we can only cancel an expression that is a *factor* of both the top and bottom. An expression is a factor of the top if it is multiplied by all the rest of the top. It is a factor of the bottom if it is multiplied by all the rest of the bottom. Here is how Sandy and Terry did the last problem on page 18.

Sandy

$$\frac{x^2+4x}{4x+16} = \frac{x\cancel{(x+4)}}{4\cancel{(x+4)}} = \frac{x}{4} \quad C$$

Right! Sandy remembered to factor first. $x + 4$ is a factor of both top and bottom.

Terry

$$\frac{x^2+\cancel{4x}}{\cancel{4x}+16} = \frac{x^2}{16} \quad X$$

Wrong! $4x$ is not a factor of either the top or the bottom.

Simplify each fraction. Remember to *factor* before you cancel.

$\frac{a^2+3a}{3a+9} =$	$\frac{3a^2}{2a+5ab} =$
$\frac{3x+5y}{6x+10y} =$	$\frac{x^2+xy}{9x} =$
$\frac{4x+8}{4x+12} =$	$\frac{2x+4}{2x+4} =$
$\frac{3a}{6a^2+9a} =$	$\frac{2a^2+2ab}{4ab} =$
$\frac{x^2+3x}{x} =$	$\frac{2x^2+6x}{2x^2+8x} =$
$\frac{x^2+x}{x^3+x} =$	$\frac{3x^2+9x}{x^2+3x} =$

Simplify each rational expression. Some are already factored for you.

$\dfrac{x^2-2x-8}{x^2+x-20} = \dfrac{\cancel{(x-4)}(x+2)}{(x+5)\cancel{(x-4)}} = \dfrac{x+2}{x+5}$	$\dfrac{x+3}{x^2+5x+6} = \dfrac{\overset{1}{\cancel{x+3}}}{\cancel{(x+3)}(x+2)} =$
$\dfrac{x^2-6x+8}{2x^2-x-6} = \dfrac{(x-2)(x-4)}{(2x+3)(x-2)} =$	$\dfrac{x-5}{x^2+x-30} = \dfrac{x-5}{(x+6)(x-5)} =$
$\dfrac{x^2+10x+25}{x^2-25} = \dfrac{(x+5)(x+5)}{(x+5)(x-5)} =$	$\dfrac{x^2-3x-10}{x-5} = \dfrac{(x-5)(x+2)}{x-5} =$
$\dfrac{x^2+6x+8}{x^2+4x+4} = \dfrac{(\quad)(\quad)}{(\quad)(\quad)} =$	$\dfrac{x^2+7x+12}{x+4} = \dfrac{(\quad)(\quad)}{x+4} =$
$\dfrac{x^2-16}{x^2+8x+16} = \dfrac{(\quad)(\quad)}{(\quad)(\quad)} =$	$\dfrac{x^2-9}{x-3} = \dfrac{(\quad)(\quad)}{x-3} =$
$\dfrac{x^2+13x+42}{x^2+2x-24} =$	$\dfrac{x^2-14x+40}{x^2-6x-40} =$
$\dfrac{x^2-16}{x^2+11x+28} =$	$\dfrac{x^2-4x-60}{x^2-4x-60} =$
$\dfrac{4x^2-25}{2x^2-3x-5} =$	$\dfrac{3x+1}{3x^2-14x-5} =$

Simplify each fraction.

$\frac{a^2+2a}{a^2+3a+2} = \frac{a\cancel{(a+2)}}{\cancel{(a+2)}(a+1)} = \frac{a}{a+1}$	$\frac{3x-3}{x^2+2x-3} =$
$\frac{x^2-11x-12}{x^2-12x} =$	$\frac{x^2+5x}{x^2+8x+15} =$
$\frac{2x^2-14x}{x^2-10x+21} =$	$\frac{5a^2+10a}{a^2-7a-18} =$
$\frac{(x+2)^2}{x^2-4} = \frac{\cancel{(x+2)}(x+2)}{\cancel{(x+2)}(x-2)} = \frac{x+2}{x-2}$	$\frac{(x+6)^2}{x^2-36} =$
$\frac{(x-5)^2}{x^2-11x+30} =$	$\frac{2x^2-3x-20}{(2x+5)^2} =$
$\frac{4x-6}{(2x-3)^2} =$	$\frac{3a^2-12a}{(a-4)^2} =$

To completely simplify each fraction below you have to factor more than once.

$$\frac{3a^2+15a}{a^3-25a} = \frac{3\cancel{a}(a+5)}{\cancel{a}(a^2-25)} = \frac{3\cancel{(a+5)}}{\cancel{(a+5)}(a-5)} = \frac{3}{a-5}$$

$$\frac{4a-12}{2a^2-18} =$$

$$\frac{2x^2-2x}{2x^3+6x^2-8x} =$$

$$\frac{x^3-3x^2-10x}{7x^2-35x} =$$

Dividing Polynomials

Dividing one polynomial by another can often be done by rewriting the problem as a rational expression and then simplifying it. Here are some examples:

Divide.

$$(2x^2 + 10x) \div 2x = \frac{2x^2 + 10x}{2x} = \frac{\overset{1}{\cancel{2x}}(x+5)}{\underset{1}{\cancel{2x}}} = x + 5$$

$$(5a^2 - 10a) \div 5a =$$

$$(x^2 + x - 20) \div (x - 4) =$$

$$(3x^2 + 7x + 4) \div (x + 1) =$$

$$(x^3 - 16x) \div (x + 4) =$$

If the trinomial in the last problem had been $3x^2 + 7x + 6$, we couldn't have factored it. Then this method of dividing wouldn't have worked and we would have had to use long division. Long division of polynomials is a lot like long division of whole numbers.

First you make sure that the terms are arranged in order by their exponents, with the largest exponent first.

$$x+1\overline{\smash{)}\,3x^2 + 7x + 6}$$

Then you divide the first term of the divisor (x) into the first term of the dividend ($3x^2$) and write the result ($3x$) over the term it matches.

$$\begin{array}{r@{}l} & \;3x \\ x+1 & \overline{\smash{)}\,3x^2 + 7x + 6} \end{array}$$

Next you multiply the result by the whole divisor, write the answer under the dividend, and subtract.

$$\begin{array}{r@{}l} & \;3x \\ x+1 & \overline{\smash{)}\,3x^2 + 7x + 6} \\ & \;\underline{3x^2 + 3x} \\ & \qquad\;\; 4x \end{array}$$

Bring down the next term and repeat the last two steps until you are finished. If there is a remainder, write it over the divisor and add this to the quotient.

$$\begin{array}{r@{}l} & \;3x + 4 + \frac{2}{x+1} \\ x+1 & \overline{\smash{)}\,3x^2 + 7x + 6} \\ & \;\underline{3x^2 + 3x} \\ & \qquad\;\; 4x + 6 \\ & \qquad\;\; \underline{4x + 4} \\ & \qquad\qquad\;\; 2 \end{array}$$

Finish each division problem. Check each answer with your teacher before doing the next problem. Be sure to ask for help if you need it.

$$\begin{array}{r}
x \\
x+2\,\overline{)\,x^2+9x+18} \\
\underline{x^2+2x} \\
7x
\end{array}$$

$$\begin{array}{r}
2x \\
x-3\,\overline{)\,2x^2+11x-4} \\
\underline{2x^2-6x} \\
17x
\end{array}$$

Remember, to subtract a polynomial, we add its opposite.

$$\begin{array}{r}
3x \\
x-4\,\overline{)\,3x^2-10x+17}
\end{array}$$

$$\begin{array}{r}
3x^2 \\
2x+3\,\overline{)\,6x^3-x^2-7x+12} \\
\underline{6x^3+9x^2}
\end{array}$$

Here are some division problems for you to do by yourself.

$(x^2+7x+19) \div (x+4)$

$(5x^2+x+30) \div (5x+6)$

$(2x^2+3x+11) \div (x-2)$

$(3x^3+5x^2-2x-4) \div (x+1)$

Rewriting Fractions in Simplest Form

Here is how Sandy and Terry simplified $\frac{8}{12}$:

Sandy

$$\frac{\overset{4}{\cancel{8}}}{\underset{6}{\cancel{12}}} = \frac{4}{6}$$

2 goes into both 8 and 12.
2 into 8 is **4**.
2 into 12 is **6**.

Terry

$$\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}} = \frac{2}{3}$$

4 is the biggest number that goes into 8 and 12.
4 into 8 is **2**.
4 into 12 is **3**.

Both Sandy and Terry simplified $\frac{8}{12}$, but only Terry rewrote it in **simplest form**. That's because Terry factored out the *biggest* number that goes into 8 and 12, while Sandy factored out a smaller number.

Here's how Sandy finished simplifying $\frac{8}{12}$:

$$\frac{\overset{4}{\cancel{8}}}{\underset{6}{\cancel{12}}} = \frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{6}}} = \frac{2}{3}$$

2 goes into both 4 and 6.
2 into 4 is **2**.
2 into 6 is **3**.

After simplifying $\frac{8}{12}$ as much as possible, both Sandy and Terry ended up with $\frac{2}{3}$. You can see that Terry's way was shortest. Terry factored out the *biggest* number that goes into the top and bottom of the fraction, so it took only one step to find the simplest form.

Rewrite each fraction in simplest form.

$\frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{12}}} = \frac{5}{3}$ — **4** goes into both 20 and 12. 4 into 20 is **5**. 4 into 12 is **3**.		$\frac{30}{20} =$	$\frac{8}{20} =$
$\frac{18}{24} =$	$\frac{16}{20} =$	$\frac{-16}{20} =$	$\frac{12}{18} =$
$\frac{15}{60} =$	$\frac{-24}{36} =$	$\frac{60}{40} =$	$\frac{30}{90} =$
$\frac{-15}{60} =$	$\frac{32}{8} =$	$\frac{-50}{10} =$	$\frac{48}{6} =$

A rational expression is in simplest form when we have canceled all possible factors from the numerator and denominator. Write each fraction in simplest form.

9 is the biggest number that goes into 27 and 36.

$$\frac{\overset{3}{\cancel{27}}\cancel{a}}{\underset{4a^2}{\cancel{36}\cancel{a^3}}} = \frac{3}{4a^2}$$

$$\frac{48x^3y}{32x^2} =$$

$$\frac{24x^2}{32x^5} =$$

$$\frac{6ab}{20ab^2} =$$

$$\frac{-45xy}{30x^2y} =$$

$$\frac{36n^3}{48mn^2} =$$

$$\frac{15x+30}{20x} = \frac{\overset{3}{\cancel{15}}(x+2)}{\underset{4}{\cancel{20}}x} = \frac{3(x+2)}{4x}$$

$$\frac{20a-25}{35a} =$$

$$\frac{12x+30}{18x} =$$

$$\frac{16x+8}{28x^2} =$$

$$\frac{8x+12}{12x+16} =$$

$$\frac{9x+45}{18x-36} =$$

$$\frac{2a^2+4a}{6a^3-6a^2} =$$

$$\frac{6x^3y-24xy}{4x^3-8x^2} =$$

$$\frac{4x^2+4x-8}{10x^2-40x+30} =$$

$$\frac{6x^3+18x^2-60x}{9x^2y-36xy+36y} =$$

Simplifying Multiplication Problems

When we are multiplying fractions it's usually easier to simplify *before* we multiply rather than after. That way we don't have to deal with such big numbers and complicated polynomials. Here's how Sandy and Terry did the same problem:

Sandy simplified *after* multiplying: $\frac{7}{12} \cdot \frac{17}{21} = \frac{\overset{17}{\cancel{119}}}{\underset{36}{\cancel{252}}} = \frac{17}{36}$

I'll have to try lots of numbers to see if this can be simplified.
2 goes into 252 but not 119.
3 goes into 252 but not 119.
4 goes into 252 but not 119.
5 doesn't go into either.
6 goes into 252 but not 119.
7 goes into both. Finally!

Terry simplified *before* multiplying: $\frac{\overset{1}{\cancel{7}}}{12} \cdot \frac{17}{\underset{3}{\cancel{21}}} = \frac{17}{36}$

Both Sandy and Terry got the same answer, but Sandy wasted a lot of time simplifying $\frac{119}{252}$. Terry's way was much easier. (Notice that Terry canceled across from the top of one fraction to the bottom of the other. This is OK in multiplication problems, because all the numerators and all the denominators are going to be multiplied together anyway.)

Multiply each pair of fractions. Simplify *before* you multiply.

7 goes into $^-7$ and 21. 5 goes into 10 and 25. $\frac{\overset{-1}{\cancel{-7}}}{\underset{5}{\cancel{25}}} \cdot \frac{\overset{2}{\cancel{10}}}{\underset{3}{\cancel{21}}} = \frac{-2}{15}$	$\frac{4}{7} \cdot \frac{5}{6} =$	$\frac{8}{3} \cdot \frac{9}{10} =$
$\frac{-12}{25} \cdot \frac{5}{6} =$	$\frac{3}{16} \cdot \frac{-2}{9} =$	$\frac{-1}{4} \cdot \frac{-4}{5} =$
$\frac{a}{4} \cdot \frac{3}{a} =$	$\frac{3x}{2} \cdot \frac{4}{x} =$	$\frac{y^2}{6} \cdot \frac{4}{y} =$
$\frac{a}{b} \cdot \frac{b^2}{a} =$	$\frac{x}{y^3} \cdot \frac{y}{x^2} =$	$\frac{2c}{5d} \cdot \frac{10}{c^2} =$
$\frac{5xy}{4} \cdot \frac{x^2}{y^2} =$	$\frac{5x^3}{24y^5} \cdot \frac{16y^2}{25x} =$	$\frac{8a^2}{15b^3} \cdot \frac{3b}{16a^4} =$

When the numerator and denominator of a fraction are polynomials, it makes even more sense to simplify before we multiply. The problem below is simplified *after* multiplying.

To simplify I have to factor.

$$\frac{x+2}{24x}\cdot\frac{3x}{x^2-4} = \frac{3x^2+6x}{24x^3-96x} = \frac{3x\,(x+2)}{24x\,(x^2-4)} = \frac{\overset{1}{\cancel{3x}}\,\overset{1}{\cancel{(x+2)}}}{\underset{8}{\cancel{24x}}\,\underset{1}{\cancel{(x+2)}}(x-2)} = \frac{1}{8(x-2)}$$

This can be factored again.

You can see that the effort spent in multiplying was wasted, because when we factored we got back what we started with. Here is the same problem simplified *before* multiplying.

$$\frac{x+2}{\underset{8}{\cancel{24x}}}\cdot\frac{\overset{1}{\cancel{3x}}}{x^2-4} = \frac{\cancel{x+2}}{8}\cdot\frac{1}{\cancel{(x+2)}(x-2)} = \frac{1}{8(x-2)}$$

Be smart. Simplify each problem *before* you multiply.

$$\frac{3x+6}{5x}\cdot\frac{x+4}{x^2+2x} =$$

$$\frac{8a}{a^2-16}\cdot\frac{a+4}{4} =$$

$$\frac{x^2-1}{3}\cdot\frac{2}{x^2-x} =$$

$$\frac{x^2-x-12}{x^2}\cdot\frac{x}{x-4} =$$

$$\frac{x^2+5x+6}{x^2-1}\cdot\frac{x+1}{x+2} =$$

$$\frac{x^2+6x+8}{x^2-4x+3}\cdot\frac{x^2-5x+4}{5x+10} =$$

Do you think multiplication of rational expressions is commutative and associative? Make a guess after you do these problems. (Remember to simplify before you multiply.)

$$\frac{2}{3}\cdot\frac{-3}{8} =$$

$$\frac{-3}{8}\cdot\frac{2}{3} =$$

$$14\cdot\frac{-5}{7} =$$

$$\frac{-5}{7}\cdot 14 =$$

$$\frac{x^2}{3y}\cdot\frac{y}{4x} =$$

$$\frac{y}{4x}\cdot\frac{x^2}{3y} =$$

$$\frac{x+4}{x-3}\cdot\frac{2x-6}{3x+12} =$$

$$\frac{2x-6}{3x+12}\cdot\frac{x+4}{x-3} =$$

$$\frac{x^2-64}{x+2}\cdot\frac{x^2+4x+4}{x^2+7x-8} =$$

$$\frac{x^2+4x+4}{x^2+7x-8}\cdot\frac{x^2-64}{x+2} =$$

$$\left(\frac{1}{3}\cdot\frac{4}{5}\right)\cdot\frac{9}{2} = \frac{\cancel{4}^{\,2}}{\cancel{15}_{\,5}}\cdot\frac{\cancel{9}^{\,3}}{\cancel{2}_{\,1}} = \frac{6}{5}$$

$$\frac{1}{3}\left(\frac{\cancel{4}^{\,2}}{5}\cdot\frac{9}{\cancel{2}_{\,1}}\right) = \frac{1}{\cancel{3}_{\,1}}\cdot\frac{\cancel{18}^{\,6}}{5} = \frac{6}{5}$$

$$\left(\frac{1}{x}\cdot\frac{3}{y}\right)\cdot\frac{y}{x} =$$

$$\frac{1}{x}\left(\frac{3}{y}\cdot\frac{y}{x}\right) =$$

$$\left(\frac{3}{8}\cdot\frac{2}{9}\right)\cdot\frac{5}{4} =$$

$$\frac{3}{8}\left(\frac{2}{9}\cdot\frac{5}{4}\right) =$$

$$5\left(\frac{1}{2}\cdot\frac{x}{9}\right) =$$

$$\left(5\cdot\frac{1}{2}\right)\cdot\frac{x}{9} =$$

$$\left(\frac{x+3}{x-2}\cdot\frac{x-3}{x+2}\right)\frac{1}{x} =$$

$$\frac{x+3}{x-2}\left(\frac{x-3}{x+2}\cdot\frac{1}{x}\right) =$$

If you guessed that multiplication of rational expressions is commutative and associative you were right. (A few examples aren't enough to prove this, but it *is* true.) The Distributive Principle works for rational expressions, too. Use it to do these problems.

$12\left(\frac{1}{3}+\frac{1}{2}\right)=\cancel{12}^{4}\cdot\frac{1}{\cancel{3}}+\cancel{12}^{6}\cdot\frac{1}{\cancel{2}}$
$=4+6=10$

$20\left(\frac{1}{5}+\frac{3}{10}\right)=$

$18\left(\frac{1}{2}-\frac{2}{9}\right)=$

$24\left(\frac{x}{3}+\frac{x}{8}\right)=$

$18\left(\frac{x}{9}+\frac{4x}{6}\right)=$

$x\left(\frac{3}{x}+\frac{5}{x}\right)=$

$x^2\left(\frac{7}{x}-\frac{4}{x}\right)=$

$12x\left(\frac{1}{2x}+\frac{3}{4x}\right)=$

$20x\left(\frac{1}{10x}-\frac{1}{5x}\right)=$

$x^2\left(\frac{4}{x}+\frac{3}{x^2}\right)=$

$x^2\left(\frac{2}{x^2}+\frac{6}{x}\right)=$

$5x^2\left(\frac{3}{5}+\frac{2}{x^2}\right)=$

$12a\left(\frac{5}{3a}-1\right)=$

$4a^2\left(\frac{4}{a}+\frac{1}{2a^2}\right)=$

$abc\left(\frac{1}{ab}+\frac{1}{bc}\right)=$

$100x\left(\frac{1}{20}-\frac{3}{x}\right)=$

Reciprocals

Every number except 0 has a **reciprocal**. When we multiply a number by its reciprocal we always get 1.

"The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$." $\quad \frac{2}{3} \cdot \frac{3}{2} = 1$

"The reciprocal of $\frac{1}{8}$ is $\frac{8}{1}$." $\quad \frac{1}{8} \cdot \frac{8}{1} = 1$

"$\frac{-4}{3}$ is the reciprocal of $\frac{3}{-4}$." $\quad \frac{-3}{4} \cdot \frac{4}{-3} = 1$

"$\frac{9}{x+1}$ is the reciprocal of $\frac{x+1}{9}$." $\quad \frac{x+1}{9} \cdot \frac{9}{x+1} = 1$

Do you see a rule for finding a reciprocal?

Find the reciprocal of each fraction.

Fraction	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{7}{2}$	$\frac{2}{7}$	$\frac{-5}{3}$	$\frac{x}{4}$	$\frac{6}{y}$	$\frac{5a}{2}$	$\frac{1}{3x}$	$\frac{x-4}{3}$	$\frac{x+1}{x-1}$
Reciprocal											

Complete each sentence below.

$10 = \frac{10}{1}$, so the reciprocal of 10 is $\frac{1}{10}$.	$y =$, so the reciprocal of y is .
$-2 =$, so the reciprocal of -2 is .	$-a =$, so the reciprocal of $-a$ is .
$4\frac{2}{3} =$, so the reciprocal of $4\frac{2}{3}$ is .	$x + 3 =$, so the reciprocal of $x + 3$ is .
$0.5 =$, so the reciprocal of 0.5 is .	$5xy =$, so the reciprocal of $5xy$ is .
$1.3 =$, so the reciprocal of 1.3 is .	$x^2 - 4 =$, so the reciprocal of $x^2 - 4$ is .

To find the reciprocal of a number, write the number as a fraction and then invert it (turn it upside down).

Dividing Fractions

When we divide an integer by another integer we get the same answer as when we *multiply* the first integer by the *reciprocal* of the second integer. Here are two examples:

$$\begin{cases} 12 \div 4 = 3 \\ 12 \cdot \frac{1}{4} = \frac{12}{1} \cdot \frac{1}{4} = 3 \end{cases} \qquad \begin{cases} -18 \div 2 = -9 \\ -18 \cdot \frac{1}{2} = \frac{-18}{1} \cdot \frac{1}{2} = -9 \end{cases}$$

The same thing is true when we divide a fraction by a fraction. We can get the answer by multiplying the first fraction by the reciprocal of the second fraction. So when we divide fractions we first have to rewrite the problem:

The division sign gets changed to a multiplication sign.
And the second fraction gets replaced by its reciprocal.

Then we multiply to get the answer.

Do each division problem by changing it to multiplication.

$\frac{6}{7} \div \frac{5}{8} = \frac{6}{7} \cdot \frac{8}{5} = \frac{48}{35}$	$\frac{1}{3} \div \frac{1}{2} =$
$\frac{-3}{10} \div \frac{2}{7} =$	$\frac{5}{2} \div 3\frac{1}{3} =$
$1\frac{3}{4} \div \frac{5}{2} =$	$8 \div \frac{2}{5} =$
$\frac{3x}{5} \div \frac{4}{x} =$	$\frac{8a}{5} \div \frac{b}{a} =$
$\frac{x^2}{y} \div \frac{x^2}{y} =$	$\frac{2a}{5b} \div 3b^2 =$
$\frac{a^2b^3}{4} \div \frac{7}{ab} =$	$\frac{x}{3a} \div \frac{y}{4b} =$

$\frac{3}{x} \div \frac{x+1}{x-2} =$

$(a-5) \div \frac{a+2}{8} =$

$\frac{x+6}{x-2} \div \frac{x+2}{x-6} =$

Each division problem below can be simplified. First rewrite the problem as a multiplication problem. Then simplify before multiplying.

$\frac{3}{2} \div \frac{1}{4} = \frac{3}{\cancel{2}_{1}} \cdot \frac{\cancel{4}^{2}}{1} = \frac{6}{1} = 6$	$\frac{-5}{6} \div \frac{35}{36} =$
$\frac{8}{9} \div 24 =$	$15 \div \frac{5}{3} =$
$\frac{3}{2x} \div \frac{6}{5} =$	$\frac{4x}{3} \div \frac{x^2}{6} =$
$\frac{-9a}{4b} \div \frac{a^2}{b^2} =$	$\frac{7y}{x^2} \div \frac{4y}{3x^2} =$
$\frac{x^2y^2}{6} \div 2xy =$	$4ab^2 \div \frac{12a}{7b^2} =$

$$\frac{3x+12}{x^2} \div \frac{x+4}{x} = \frac{3x+12}{x^2} \cdot \frac{x}{x+4} = \frac{3\cancel{(x+4)}^{1}}{\cancel{x^2}_{x}} \cdot \frac{\cancel{x}^{1}}{\cancel{x+4}_{1}} = \frac{3}{x}$$

$$\frac{2a+6}{a^3} \div \frac{a+3}{a} =$$

$$\frac{10x}{3x-9} \div \frac{10x^2}{3x^2-9x} =$$

$$\frac{x^2+3x-10}{x^2+3x} \div \frac{x^2-4x+4}{4x+12} =$$

!

Some of these are multiplication problems and some are division problems. Be sure you look closely before you start each one. Always simplify if you can.

$\frac{9}{14} \div \frac{15}{8} =$	$\frac{-3}{8} \cdot \frac{-4}{9} =$	$\frac{20}{21} \div \frac{7}{10} =$
$\frac{28x}{33y} \div \frac{8x}{3y} =$	$\frac{14n^3}{9t} \cdot \frac{21t^2}{2n} =$	$\frac{6a}{7b^2} \div 42ab^2 =$

$\frac{2}{x+1} \cdot \frac{x-5}{3} =$	$\frac{2}{x+1} \div \frac{x-5}{3} =$
$\frac{5}{8} \cdot \frac{5x^2}{8x-1} =$	$\frac{5}{8} \div \frac{5x^2}{8x-1} =$
$(x-10) \cdot \frac{x-10}{x+10} =$	$(x-10) \div \frac{x-10}{x+10} =$
$\frac{x+3}{6x} \cdot \frac{x-1}{2x(x+3)} =$	$\frac{x+3}{6x} \div \frac{x-1}{2x(x+3)} =$

$\frac{x^2+8x}{9x} \div \frac{x^2-64}{3x^2} =$

$\frac{x^2-xy}{y} \cdot \frac{y^2}{2x-2y} =$

$\frac{x^2+5x-14}{x^2-4x-5} \cdot \frac{(x+1)^2}{x^2-49} =$

Each problem on this page has a simple answer. Work carefully and take your time.

$$\frac{25a^3b}{12c^5} \cdot \frac{9abc}{35d^4} \div \frac{15a^4b^2}{14c^4d^4} =$$

$$\left(3\tfrac{1}{2}\right)\left(3\tfrac{1}{3}\right)\left(2\tfrac{2}{5}\right)\left(4\tfrac{3}{8}\right)\left(2\tfrac{2}{11}\right)\left(1\tfrac{4}{7}\right) =$$

$$\frac{3x^2 + 21x}{x^2 - 49} \cdot \frac{x^2 - x}{6x^3 - 9x^2} \cdot \frac{4x^2 - 9}{3x - 3} =$$

$$\frac{3x^2y^2 + 9xy^2 - 30y^2}{4x^3 + 4x^2 - 168x} \cdot \left(\frac{5x^2 - 245}{3x^2y - 75y} \div \frac{5x^2y - 45xy + 70y}{6x^3 - 66x^2 + 180x}\right) =$$

Written Work

Do these problems on some clean paper. Label each page of your work with your name, your class, the date, and the book number. Also number each problem. Keep this written work inside your book, and turn it in with your book when you are finished. Please do a neat job.

1. Why can't we substitute 3 for x in the rational expression $\frac{2}{x-3}$?

2. Write the condition for each rational expression. Then write any numbers which may *not* be substituted for x.

   $\frac{12}{x}$ $\quad$ $\frac{1}{x-4}$ $\quad$ $\frac{3}{2x}$ $\quad$ $\frac{^-6}{x+1}$ $\quad$ $\frac{x+4}{2x-7}$

3. Write each rational number as a fraction. Then write its reciprocal. Arrange your answers in a table.

   $^-1000$ $\quad$ 0.9 $\quad$ 0.03 $\quad$ 0.237 $\quad$ $11\frac{1}{2}$ $\quad$ $5\frac{2}{3}$ $\quad$ $^-2.7$

4. Simplify as many of these rational expressions as you can. If an expression cannot be simplified, write "already in simplest form" next to it.

   $\frac{4x}{2x}$ $\quad$ $\frac{4x-2}{2x}$ $\quad$ $\frac{x+2}{2x}$ $\quad$ $\frac{x^2+4x}{2x}$ $\quad$ $\frac{x^2+4}{x+2}$ $\quad$ $\frac{x^2-4}{x-2}$ $\quad$ $\frac{x^2-4}{x+2}$

5. Multiply each expression by $\frac{x}{x-1}$. Be sure your answer is in simplest form.

   $\frac{3}{8}$ $\quad$ $\frac{^-2}{x}$ $\quad$ $\frac{x}{x-1}$ $\quad$ $\frac{x-1}{x}$ $\quad$ $\frac{x^2-1}{x^2+1}$ $\quad$ $\frac{x+5}{x-2}$

6. Divide each expression by $\frac{x}{y^2}$. Be sure your answer is in simplest form.

   $\frac{^-1}{3}$ $\quad$ $\frac{x}{y^2}$ $\quad$ $\frac{y^2}{x}$ $\quad$ $\frac{2y}{7x}$ $\quad$ $\frac{x^2+3x}{5y^2}$ $\quad$ $\frac{x}{xy^2-5y^2}$

7. Copy and complete each statement.

   Multiplying by $\frac{1}{7}$ gives the same answer as dividing by ________.

   Dividing by 4 gives the same answer as multiplying by ________.

8. Use the statements in problem 7 to write each of these expressions another way.

   $\frac{1}{7}(3x-2)$ $\quad$ $\frac{1}{7}a^2b^2$ $\quad$ $\frac{xy^3}{4}$ $\quad$ $\frac{x^2-1}{4}$

9. Rewrite each expression as a multiplication problem. Then use the Distributive Principle to multiply. $\quad$ $\frac{3x+15}{3}$ $\quad$ $\frac{2y-20}{^-4}$ $\quad$ $\frac{6a+27}{^-3}$ $\quad$ $\frac{5x^2+40x}{5x}$

10. Is there a Distributive Principle for division? Explain.